Exercise A

QUESTION 9

9.1 Given: P(A) = 0,6
P(B) = 0,3
P(A or B) = 0,8   where A and B are two different events

Are the events A and B mutually exclusive? Justify your answer with appropriate calculations and/or a diagram.

9.2 The table below shows data on the monthly income of employed people in two residential areas. Representative samples were used in the collection of the data.

MONTHLY INCOME (IN RANDS)	AREA 1	AREA 2	TOTAL
$x < 3\ 200$	500	460	960
$3\ 200 \leq x < 25\ 600$	1 182	340	1 522
$x \geq 25\ 600$	150	14	164
Total	1 832	814	2 646

9.2.1 What is the probability that a person chosen randomly from the entire sample will be:

(a) From Area 1

(b) From Area 2 and earn less than R3 200 per month

(c) A person from Area 2 who earns more than or equal to R3 200

9.2.2 Prove that earning an income of less than R3 200 per month is not independent of the area in which a person resides.

9.2.3 Which is more likely: a person from Area 1 earning less than R3 200 or a person from Area 2 earning less than R3 200? Show calculations to support your answer.

Solution

9.1 For mutually exclusive events:

$$P(A \text{ and } B) = 0$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$0,8 = 0,6 + 0,3 - P(A \text{ and } B)$$

$$P(A \text{ and } B) = 0,9 - 0,8$$

$$P(A \text{ and } B) = 0,1$$

∴ A and B are not mutually exclusive

∴ $P(A \text{ and } B) \neq 0$

Exercise A

QUESTION 9

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(c) A person from Area 2 who earns more than or equal to R3 200

9.2.2 Prove that earning an income of less than R3 200 per month is not independent of the area in which a person resides.

9.2.3 Which is more likely: a person from Area 1 earning less than R3 200 or a person from Area 2 earning less than R3 200? Show calculations to support your answer.

9.2.1 (a) $\dfrac{1832}{2646} = 0,69$

(b) $\dfrac{460}{2646} = 0,17$

(c) $\dfrac{340 + 14}{2646} = 0,13$

9.2.2 For Independent Events: $P(A \text{ and } B) = P(A) \times P(B)$

$P(x < 3200 \text{ and From Area 1}) = \dfrac{500}{2646} = 0,19$

$P(x < 3200) \times P(\text{From Area 1}) = \dfrac{960}{2646} \times \dfrac{1832}{2646} = 0,25$

$P(x < 3200 \text{ and From Area 1}) \neq P(x < 3200) \times P(\text{From Area 1})$

∴ The events are not independent.

Exercise A

QUESTION 9

9.1 Given:
$P(A) = 0,6$
$P(B) = 0,3$
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9.2.2 Prove that earning an income of less than R3 200 per month is not independent of the area in which a person resides.

9.2.3 Which is more likely: a person from Area 1 earning less than R3 200 or a person from Area 2 earning less than R3 200? Show calculations to support your answer.

9.2.3 $P(\text{From Area 1 and } x < 3200) = \dfrac{500}{1832} = 0,27$

$P(\text{From Area 2 and } x < 3200) = \dfrac{460}{814} = 0,57$

∴ Person from Area 2 earning less than 3200 is more likely.

Exercise B

QUESTION 9

9.1 Given: $P(A) = 0.2$
$P(B) = 0.5$
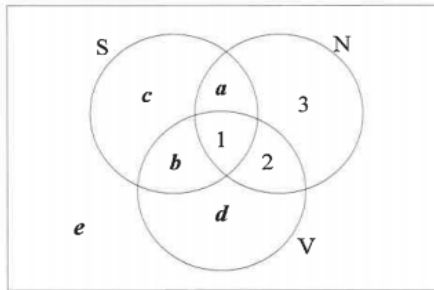$P(A \text{ or } B) = 0.6$ where A and B are two different events

9.1.1 Calculate $P(A \text{ and } B)$.

9.1.2 Are the events A and B independent ? Show your calculations.

9.2 A survey was conducted amongst 100 learners at a school to establish their involvement in three codes of sport, soccer, netball and volleyball. The results are shown below.

- 55 learners play soccer (S)
- 21 learners play netball (N)
- 7 learners play volleyball (V)
- 3 learners play netball only
- 2 learners play soccer and volleyball
- 1 learner plays all 3 sports

The Venn diagram below shows the information above.

9.2.1 Determine the values of a, b, c, d and e.

9.2.2 What is the probability that one of the learners chosen at random from this group plays netball or volleyball?

Solution

9.1.1 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$0.6 = 0.2 + 0.5 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.7 - 0.6$

$\therefore P(A \text{ and } B) = 0.1$

9.1.2 For Independent events: $P(A \text{ and } B) = P(A) \times P(B)$

$P(A) \times P(B) = 0.2 \times 0.5$

$= 0.1$

$\therefore P(A \text{ and } B) = P(A) \times P(B)$

$\therefore$ The events A and B are independent

Exercise B

QUESTION 9

9.1 Given: P(A) = 0,2
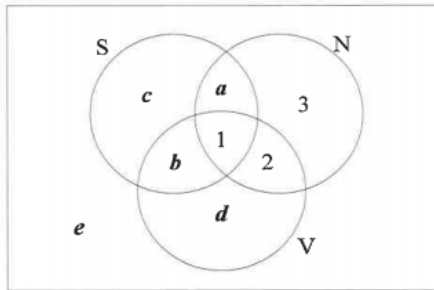P(B) = 0,5
P (A or B) = 0, 6   where A and B are two different events

9.1.1 Calculate P(A and B).

9.1.2 Are the events A and B independent ? Show your calculations.

9.2 A survey was conducted amongst 100 learners at a school to establish their involvement in three codes of sport, soccer, netball and volleyball. The results are shown below.

- 55 learners play soccer (S)
- 21 learners play netball (N)
- 7 learners play volleyball (V)
- 3 learners play netball only
- 2 learners play soccer and volleyball
- 1 learner plays all 3 sports

The Venn diagram below shows the information above.

9.2.1 Determine the values of *a*, *b*, *c*, *d* and *e*.

9.2.2 What is the probability that one of the learners chosen at random from this group plays netball or volleyball?

9.2.1 $a + 1 + 2 + 3 = 21$
$a = 15$

$b + 1 = 2$
$b = 1$

$1 + 1 + 2 + d = 7$
$d = 3$

$c + 15 + 1 + 1 = 55$
$c = 38$

$e + 38 + 15 + 1 + 3 + 2 + 1 + 3 = 100$
$e = 37$

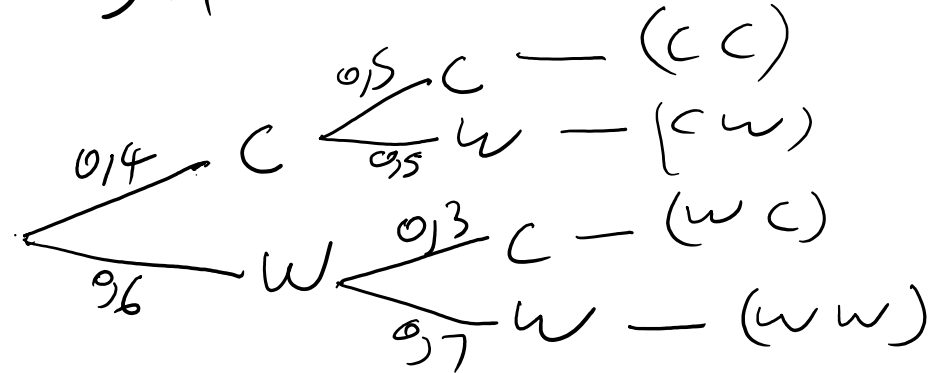9.2.2 $\dfrac{15 + 3 + 1 + 2 + 1 + 3}{100} = \dfrac{25}{100} = 0,25$

Exercise B

9.3 The probability that the first answer in a maths quiz competition will be correct is 0,4. If the first answer is correct, the probability of getting the next answer correct rises to 0,5. However, if the first answer is wrong, the probability of getting the next answer correct is only 0,3.

9.3.1 Represent the information on a tree diagram. Show the probabilities associated with each branch as well as the possible outcomes.

9.3.2 Calculate the probability of getting the second answer correct.

9.3.1

```
              0,5  C ——— (C C)
      0,4  C <
           /      0,35  W ——— (C W)
          /
         <
          \              0,3  C ——— (W C)
      0,6  W <
                  0,7  W ——— (W W)
```

9.3.2 $P(CC)$ or $P(WC)$

$= (0,4 \times 0,5) + (0,6 \times 0,3)$

$= 0,38$

Exercise C

QUESTION 8

8.1   A bag contains 3 blue marbles and 2 red marbles. A marble is taken from the bag, the colour is recorded and the marble is put aside. A second marble is taken from the bag, the colour is recorded and then put aside.

    8.1.1   Draw a tree diagram to represent the information above. Show the probabilities associated with EACH branch, as well as the possible outcomes.
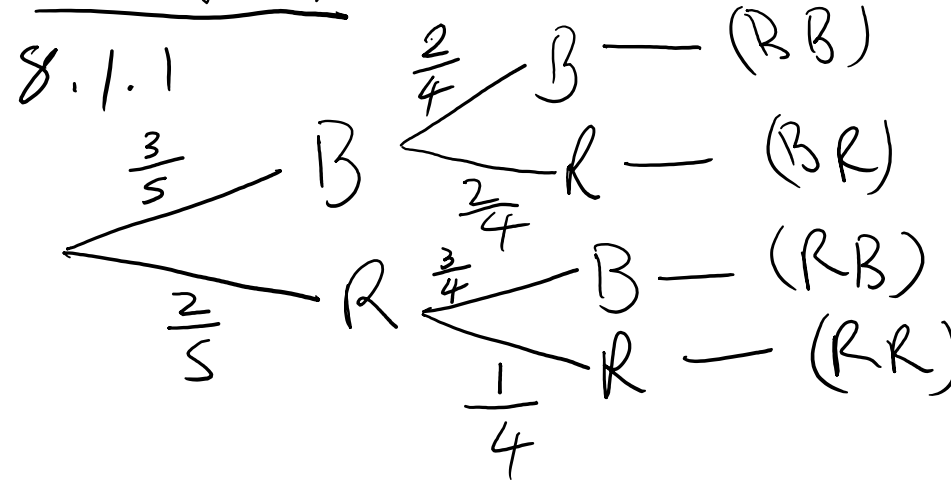
    8.1.2   Determine the probability of first taking a red marble and then taking a blue marble, in that order.

8.2   A and B are two events. The probability that event A will occur is 0,4 and the probability that event B will occur is 0,3. The probability that either event A or event B will occur is 0,58.

    8.2.1   Are events A and B mutually exclusive?
              Justify your answer with appropriate calculations.

    8.2.2   Are events A and B independent?
              Justify your answer with appropriate calculations.

Solution

8.1.1

Tree diagram:

$$\frac{3}{5} \to B \begin{cases} \frac{2}{4} \to B \to (BB) \\ \frac{2}{4} \to R \to (BR) \end{cases}$$

$$\frac{2}{5} \to R \begin{cases} \frac{3}{4} \to B \to (RB) \\ \frac{1}{4} \to R \to (RR) \end{cases}$$

8.1.2 $P(RB) = \dfrac{2}{5} \times \dfrac{3}{4} = 0{,}3$

Exercise C

QUESTION 8

8.1 A bag contains 3 blue marbles and 2 red marbles. A marble is taken from the bag, the colour is recorded and the marble is put aside. A second marble is taken from the bag, the colour is recorded and then put aside.

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    8.2.1   Are events A and B mutually exclusive?
                Justify your answer with appropriate calculations.

    8.2.2   Are events A and B independent?
                Justify your answer with appropriate calculations.

8.2.1 For mutually exclusive events: $P(A \text{ and } B) = 0$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$0,58 = 0,4 + 0,3 - P(A \text{ and } B)$$

$$P(A \text{ and } B) = 0,12$$

$\therefore$ Events A and B are not mutually exclusive

$\therefore P(A \text{ and } B) \neq 0$

8.2.2 Independent events: $P(A \text{ and } B) = P(A) \times P(B)$

$$P(A) \times P(B) = 0,4 \times 0,3$$

$$= 0,12$$

$\therefore$ Events A & B are independent

$\therefore P(A \text{ and } B) = P(A) \times P(B)$

Exercise C

QUESTION 9

A survey was done among 80 learners on their favourite sport. The results are shown below.

- 52 learners like rugby (R)
- 42 learners like volleyball (V)
- 5 learners like chess (C) only
- 14 learners like rugby and volleyball but not chess
- 12 learners like rugby and chess but not volleyball
- 15 learners like volleyball and chess but not rugby
- x learners like all 3 types of sport
- 3 learners did not like any sport

9.1 Draw a Venn diagram to represent the information above.

9.2 Show that $x = 8$.

9.3 How many learners like only rugby?

9.4 Calculate the probability that a learner, chosen randomly, likes at least TWO different types of sport.

9.1
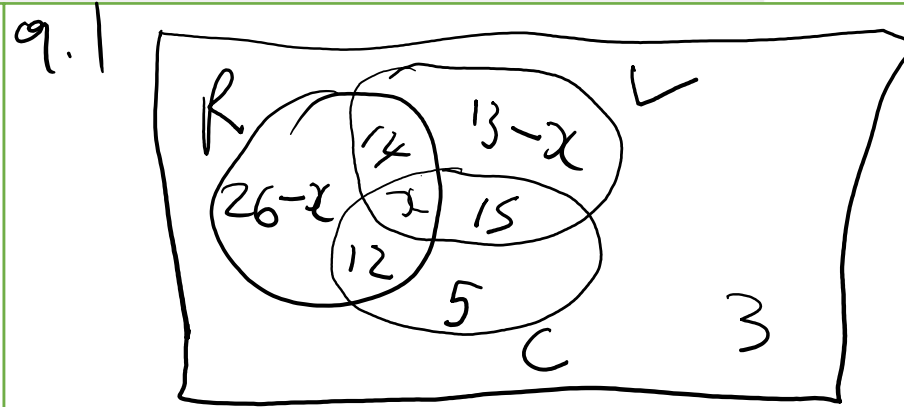
9.2 $26 - x + 14 + 13 - x + x + 12 + 15 + 5 + 3 = 80$

$$88 - x = 80$$
$$8 = x$$
$$\therefore x = 8$$

9.3 $26 - 8 = 18$

9.4 $\dfrac{12 + 14 + 8 + 15}{80} = 0{,}61$