NUMBER PATTERNS, SEQUENCES AND SERIES

2026 WORKSHEET

June 2025

QUESTION 2

2.1 Given the arithmetic series: $5 + 7 + 9 + \ldots + 93$

2.1.1 Determine the general term of the series, T_n, in the form $T_n = pn + q$. (2)

2.1.2 The given series represents the number of kilometres that an athlete ran each week in preparation for an ultramarathon. The athlete ran 93 km in the last week of the training programme. How long, in weeks, was the training programme? (2)

2.1.3 The athlete used this opportunity to raise funds for her high school. The community sponsored her R10 for each kilometre run during the training programme. Calculate the total amount that the athlete raised for her school. (3)

2.2 The general term of a geometric sequence is $T_n = 2^{n+2}$

2.2.1 Write down:

(a) The first term (1)

(b) The common ratio (1)

2.2.2 Calculate T_{20} (Write your answer as a power of 4.) (2)

2.2.3 Calculate $\sum_{n=1}^{\infty} \frac{1}{T_n}$ (3)

2.2.4 Consider the first 21 terms of the sequence $T_n = 2^{n+2}$. Calculate the sum of the terms in this sequence that are not powers of 4. (4)

[18]

QUESTION 3

Given the quadratic sequence: 14 ; 9 ; 6 ; 5 ; …

3.1 Show that the general term of this sequence is $T_n = n^2 - 8n + 21$. (3)

3.2 Two consecutive terms of the quadratic sequence have a difference of 33. Calculate the value of the larger term. (3)

3.3 The value of m is added to each term in the quadratic sequence. Determine the values of m for which only the terms between T_1 and T_7 of the quadratic sequence will have negative values. (3)

[9]

<u>November 2024</u>

QUESTION 2

2.1 The first term of an arithmetic series is 7. The common difference of this series is 5 and the series contains 20 terms.

2.1.1 Calculate the sum of this series. (2)

2.1.2 The original arithmetic series is extended to 75 terms. The sum of these 75 terms is 14 400. Using sigma notation, write down an equation for the sum of the terms added to the original series. (4)

2.2 The sequence of the first differences of a quadratic pattern is: 1 ; 3 ; 5 ; …

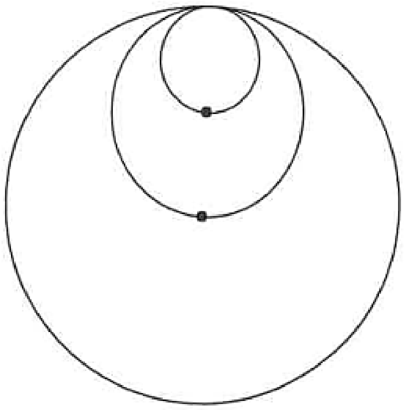
2.2.1 If T_{99} of this quadratic pattern is 9 632, calculate the value of T_{98}. (3)

2.2.2 If it is further given that the third term of the quadratic pattern is 32, determine the general term, T_n, of the quadratic pattern. (5)

[14]

QUESTION 3

A circle with radius 6 cm is drawn.
A second, smaller circle is drawn through the centre of the first circle and also touches the first circle internally, as shown in the diagram.
A third, smaller circle is drawn through the centre of the second circle and touches the second circle internally. The process of drawing circles continues and forms a geometric pattern.



3.1 Write down the radius of the 3^{rd} circle. (2)

3.2 Calculate the sum of the areas of the first 10 circles. (4)

3.3 Which circle has a diameter of $\frac{3}{128}$ cm? (4)

[10]

NUMBER PATTERNS, SEQUENCES AND SERIES

2026 WORKSHEET SOLUTIONS

June 2025

QUESTION/*VRAAG* 2

2.1.1	$T_n = 2n + 3$
2.1.2	$93 = 2n + 3$ $90 = 2n$ $45 = n$
2.1.3	$50 + 70 + 90 + ... + 930$ $S_{45} = \frac{45}{2}[2(50) + (45-1)(20)]$ $S_{45} = R22\,050$ Total raised = R22 050 **OR/*OF*** $50 + 70 + 90 + ... + 930$ $S_{45} = \frac{45}{2}[50 + 930]$ $S_{45} = R22\,050$ Total raised = R22 050 **OR/*OF*** $5 + 7 + 9 + ... + 93$ $S_{45} = \frac{45}{2}[2(5) + (45-1)(2)]$ $S_{45} = 2\ 205\text{km}$ $S_{45} = R22\,050$ Total raised = R22 050

2.2.1 a)	$T_1 = (2)^{1+2}$ $T_1 = 8$
2.2.1 b)	$r = 2$
2.2.2	$T_{20} = (2)^{20+2}$ $T_{20} = 2^{22} = (2^2)^{11}$ $= 4^{11}$

2.2.3	$\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\dots$ $S_\infty=\frac{a}{1-r}$ $=\frac{\frac{1}{8}}{1-\frac{1}{2}}$ $\therefore S_\infty=\frac{1}{4}$
2.2.4	$8+4^2+32+4^3+\dots+4^{11}+\dots$ $S_{21}-S_{10}=\frac{8(2^{21}-1)}{2-1}-\frac{16(4^{10}-1)}{4-1}$ $=16\ 777\ 208-5\ 592\ 400$ $=11\ 184\ 808$ **OR/*OF*** $8+32+128+$ $S_{11}=\frac{8(4^{11}-1)}{4-1}$ $\therefore S_{11}=11\ 184\ 808$

QUESTION/*VRAAG* 3

3.1	$14\ ;\ 9\ ;\ 6\ ;\ 5\ ;\ \dots$ $-5\quad -3\quad -1$ $2\quad 2$ $2a=2$ $\quad 3(1)+b=-5$ $\quad 1-8+c=14$ $a=1$ $\quad b=-8$ $\quad c=21$ $\therefore T_n=n^2-8n+21$
3.2	$T_n=-5+(n-1)(2)$ $T_n=2n-7$ $2n-7=33$ $\therefore n=20$ $\therefore T_{21}=(21)^2-8(21)+21$ $T_{21}=294$ **OR/*OF*** $\therefore T_{n+1}-T_n=(n+1)^2-8(n+1)+21-n^2+8n-21$ $n^2+2n+1-8n-8+21-n^2+8n-21=33$ $2n-7=33$ $\therefore n=20$ $\therefore T_{21}=(21)^2-8(21)+21$ $T_{21}=294$

3.3	$T_7 = T_1 = 14$ $\therefore 14 + m \geq 0$ $m \geq -14$ And $T_6 = T_2$ $\therefore 9 + m < 0$ $m < -9$ $\therefore -14 \leq m < -9$

November 2024

QUESTION/*VRAAG* 2

2.1.1	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{20} = \frac{20}{2}[2(7) + (20-1)(5)]$ $S_{20} = 1\,090$
2.1.2	$T_n = 7 + (n-1)(5)$ $T_n = 5n + 2$ $S_{75} - S_{20} = 14\ 400 - 1\,090 = 13\ 310$ $\sum_{n=1}^{75}(5n+2) - \sum_{n=1}^{20}(5n+2) = 13\,310$ $\therefore \sum_{n=21}^{75}(5n+2) = 13\ 310$ **OR/*OF*** $T_{21} + T_{22} + T_{23} + ... + T_{75} = 14\,400 - 1\,090$ 107 + 112 + 117 + …. = 13 310 $T_n = 102 + 5n$ $\therefore \sum_{1}^{55}(5n+102) = 13\,310$

2.2.1	$T_n = 2n - 1$ 98th first difference = 2(98) – 1 = 195 Quadratic: $T_{98} = 9\ 632 - 195$ $= 9\ 437$

2.2.2	28 ; 29 ; 32 1 3 5 7 2 2 2 $2a = 2$ $a = 1$ $3a + b = 1$ $3(1) + b = 1$ $b = -2$ $9a + 3b + c = 32$ $T_3 = 1(3)^2 - 2(3) + c = 32$ $c = 29$ **OR/*OF*** $1 - 2 + c = 28$ $c = 29$ $\therefore T_n = n^2 - 2n + 29$

QUESTION/*VRAAG* 3

3.1	2^{nd} circle: $\frac{6}{2} = 3$ cm 3^{rd} circle: $\frac{3}{2} = 1{,}5$ cm **OR/*OF*** 3^{rd} circle $= 6\left(\frac{1}{2}\right)^2 = 1{,}5$
3.2	$36\pi ; 9\pi ; \frac{9}{4}\pi ; \ldots$ $r = \frac{1}{4}$; $a = 36\pi$ $S_{10} = \dfrac{36\pi\left(\left(\frac{1}{4}\right)^{10} - 1\right)}{\frac{1}{4} - 1}$ $S_{10} = 150{,}80$

3.3	$6\ ;\ 3\ ;\ \frac{3}{2}\ldots$ $\text{radius} = \frac{1}{2}\left(\frac{3}{128}\right) = \frac{3}{256}$ $\frac{3}{256} = 6\left(\frac{1}{2}\right)^{n-1}$ $\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$ $\left(\frac{1}{2}\right)^{9} = \left(\frac{1}{2}\right)^{n-1}$ **or/*of*** $n-1 = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{512}\right)$ $\therefore n-1=9$ $\therefore n-1=9$ $\therefore n=10$ $\therefore n=10$
	OR/*OF* $12\ ;\ 6\ ;\ 3\ ;\ \ldots$ $\text{diameter} = \frac{3}{128}$ $\frac{3}{128} = 12\left(\frac{1}{2}\right)^{n-1}$ $\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$ $\left(\frac{1}{2}\right)^{9} = \left(\frac{1}{2}\right)^{n-1}$ **or/*of*** $n-1 = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{512}\right)$ $\therefore n-1=9$ $\therefore n-1=9$ $\therefore n=10$ $\therefore n=10$