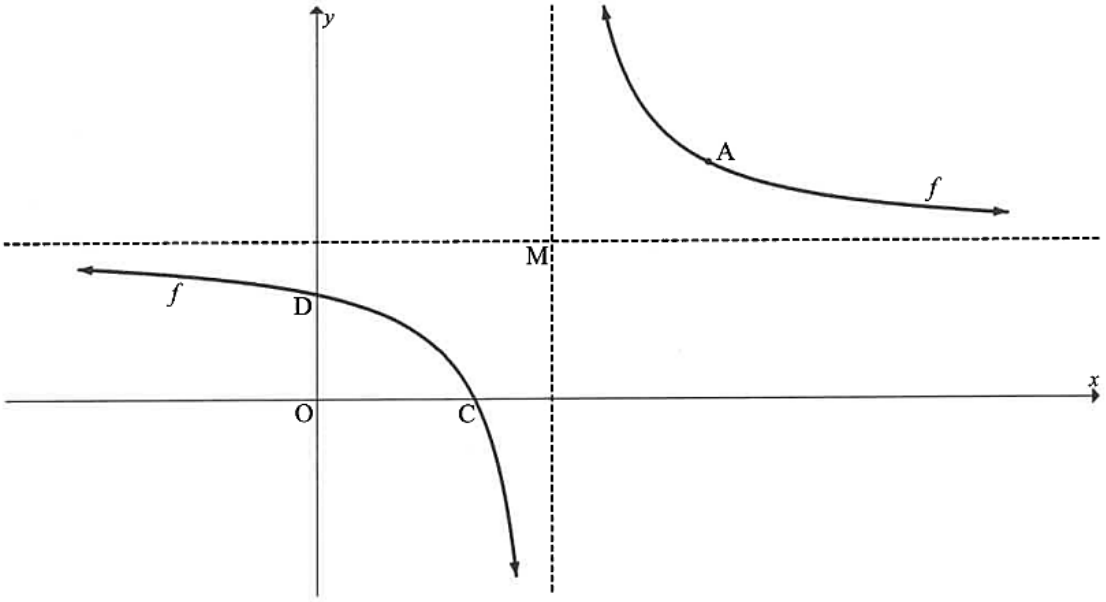
FUNCTIONS AND GRAPHS 2026

WORKSHEET

June 2025

QUESTION 4

The graph of $f(x)=\frac{4}{x-3}+4$ is drawn below. M is the point where the asymptotes of f intersect. C and D are the x- and y-intercepts respectively of f. A is the point on f that is closest to M.



4.1 Write down the coordinates of M. (2)

4.2 Calculate the coordinates of D. (2)

4.3 If $y=x+t$ is the equation of a line of symmetry of f, calculate the value of t. (2)

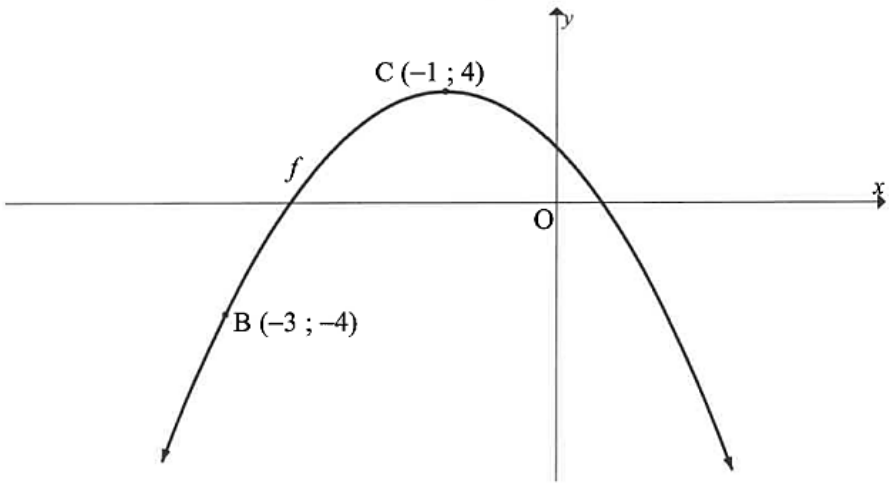
4.4 Determine the values of x for which $f(x)\leq 0$. (4)

4.5 Calculate the coordinates of A. (3)

4.6 A single transformation is applied to f to obtain a new graph defined as $h(x)=\frac{-4}{x+3}+4$. A$'$ is the image of A under this transformation. Calculate the length of AA$'$. (2)
[15]

QUESTION 5

The graph of $f(x) = a(x+p)^2 + q$ is drawn below. C(– 1 ; 4) is the turning point of f.
B(– 3 ; – 4) is a point on f.

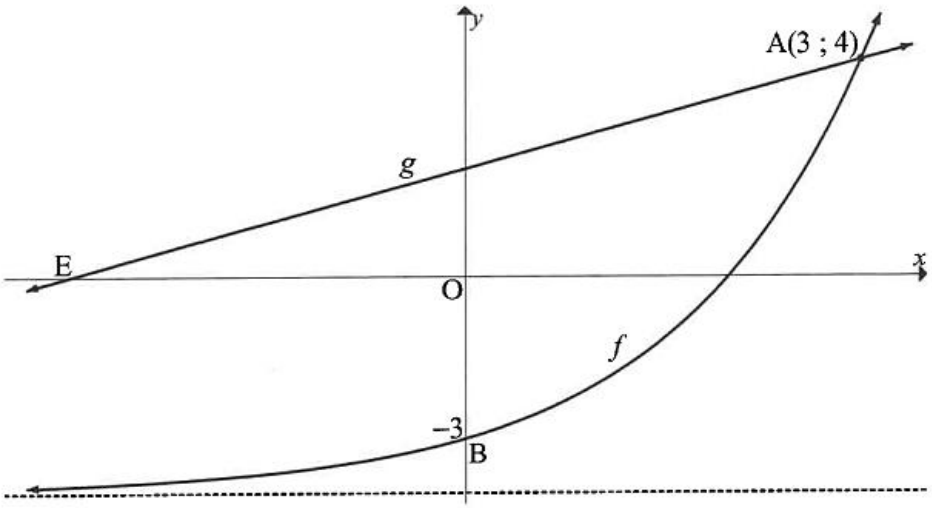


5.1 Show that $f(x) = -2x^2 - 4x + 2$. (3)

5.2 Determine the values of k for which $h(x) = f(x) + k$ will have no real roots. (2)

QUESTION 6

The graphs of $f(x) = p^x + q$ and $g(x) = mx + c$ are drawn below. A(3 ; 4) is the point of intersection of f and g. B(0 ; – 3) is the y-intercept of f. E is the x-intercept of g.



6.1 Calculate the values of p and q. (4)

6.2 Write down the range of f. (1)

6.3 The graph of g^{-1}, the inverse of g, also passes through B. Determine the equation of g in the form $y = \ldots$ (4)

6.4 Write down the equation of g^{-1} in the form $y = \ldots$ (2)
[11]

November 2024

QUESTION 4

Given: $f(x) = a^x - 1$ for $a > 0$. B$\left(2; \frac{-5}{9}\right)$ is a point on f.

4.1 Calculate the value of a. (2)

4.2 Write down the range of f. (1)

4.3 Sketch the graph of f. Clearly show the intercepts with the axes and asymptotes, if any. (3)

4.4 It is further given that C is a point on f at $y = \frac{19}{8}$.

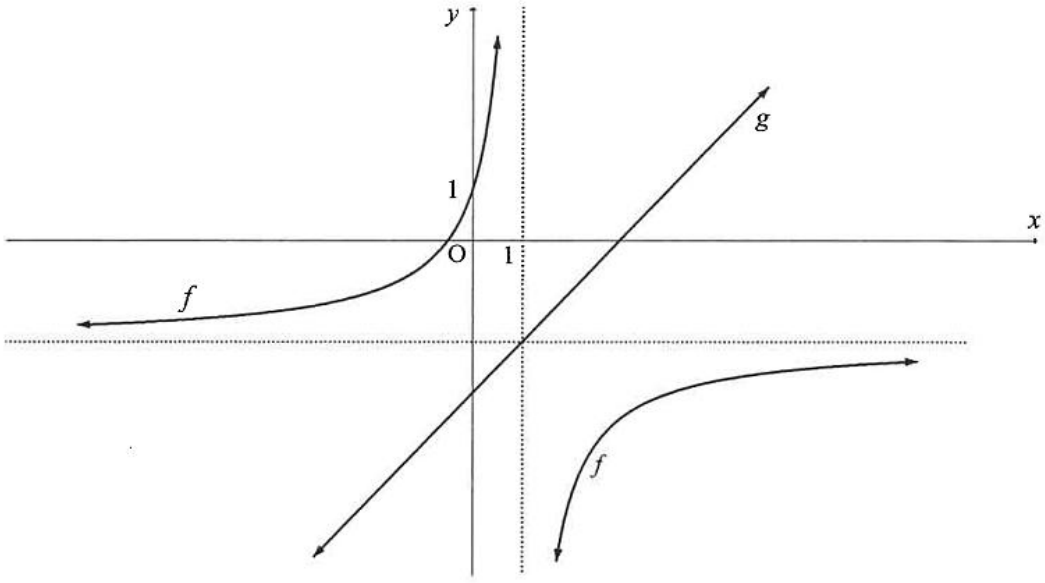
Determine the coordinates of C$'$, the image of C, when C is reflected about the line $y = x$. (3)

[9]

QUESTION 5

Sketched below is the graph of $f(x) = \frac{a}{x+p} + q$ having the domain $(-\infty; 1) \cup (1; \infty)$.

The graph of f cuts the y-axis at $(0; 1)$. A line of symmetry of f is given by $g(x) = x - 3$.



5.1 Write down the value of p. (1)

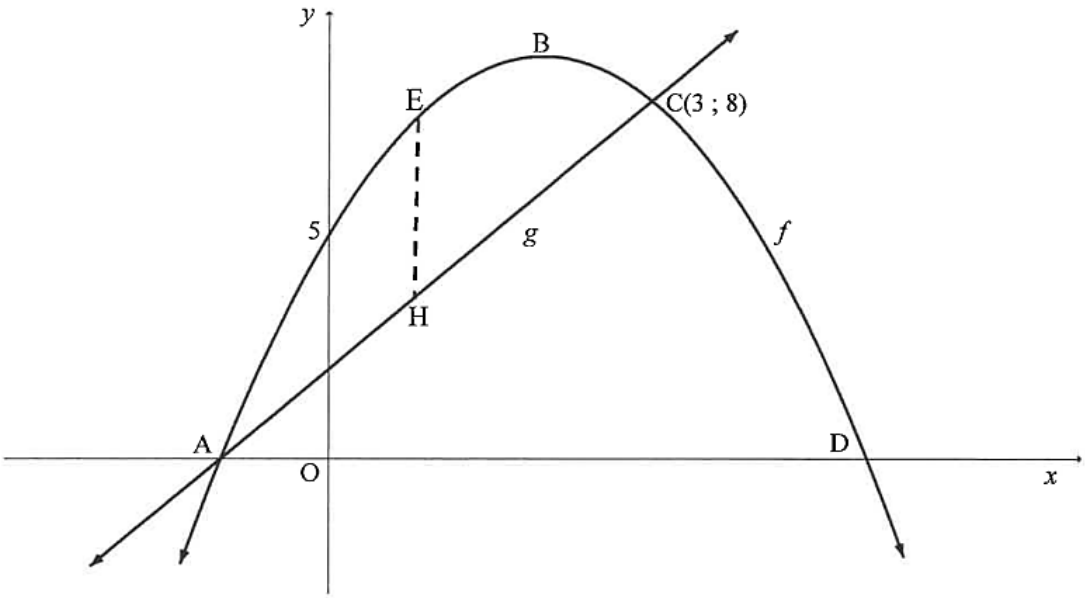
5.2 Determine the equation of the horizontal asymptote of f. (2)

5.3 Calculate the value of a. (2)

5.4 For which values of x is $f(x) \geq 0$? (3)

QUESTION 6

In the diagram below, the graphs of $f(x) = -x^2 + 4x + 5$ and g, a straight line, are drawn. $C(3\,;8)$ is a point of intersection of f and g. EH is drawn parallel to the y-axis, with E a point on f and H a point on g.



6.1 Calculate the coordinates of B, the turning point of f. (3)

6.2 Show that the equation of the line through A and C is given by $g(x) = 2x + 2$. (3)

6.3 Calculate the maximum length of EH for $f > g$. (4)

6.4 Given: $k(x) = f(x+m) = -x^2 - 2mx - m^2 + 4x + 4m + 5$

Determine the value of m such that g is a tangent to k. (5)
[15]

FUNCTIONS AND GRAPHS 2026

WORKSHEET SOLUTIONS

June 2025

QUESTION/*VRAAG* 4

4.1	M(3 ; 4)
4.2	$f(x)=\frac{4}{x-3}+4$ $y=\frac{4}{0-3}+4=\frac{8}{3}$ $\therefore \text{D}\left(0;\frac{8}{3}\right)$
4.3	M (3 ; 4) $y=x+t$ **OR/*OF*** $y=(x+p)+q$ $4=3+t$ $y=x-3+4$ $t=1$ $y=x+1$ $\therefore t=1$
4.4	$\frac{4}{x-3}+4=0$ $-4(x-3)=4$ $x-3=-1$ $x=2$ C(2 ; 0) $\therefore 2\le x<3$

4.5	$\frac{4}{x-3}+4=x+1$ $\frac{4}{x-3}=x-3$ $4=(x-3)^2$ $\pm 2=x-3$ $\therefore x=5 \quad \text{or} \quad x\neq 1$ $\therefore$ A (5 ; 6) **OR/*OF*** Point closest to the origin in $y=\frac{a}{x}$ is $(\sqrt{a};\sqrt{a})$ By translation: $A(\sqrt{a}+3;\sqrt{a}+4)$ A(5 ; 6)
4.6	$h(x)=\frac{-4}{x+3}+4$ $=\frac{4}{-x-3}+4$ $\therefore$ Reflection in $y-$ axis. $A'(-5;6)$ $AA'=10$

QUESTION/*VRAAG* 5

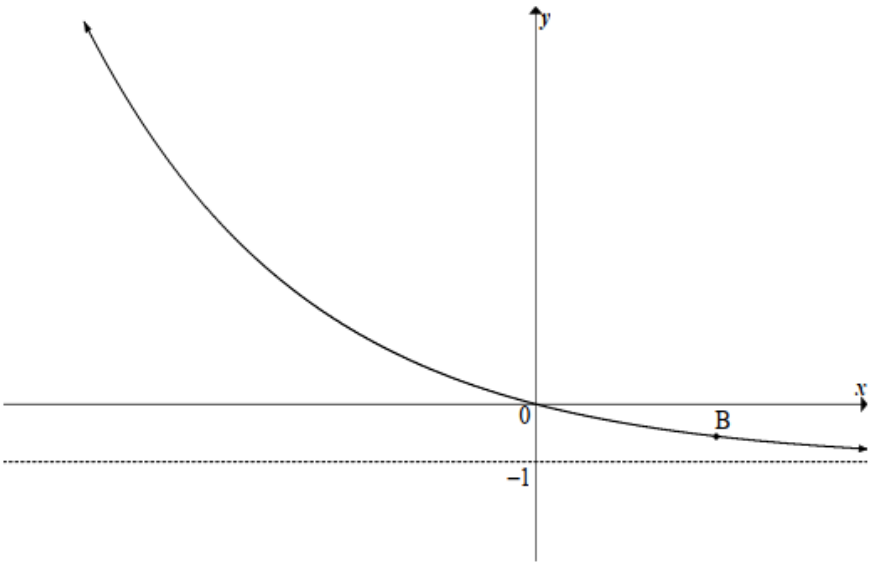
5.1	$y=a(x+1)^2+4$ $-4=a(-3+1)^2+4$ $-8=4a$ $-2=a$ $y=-2(x+1)^2+4$ $y=-2x^2-4x+2$
5.2	$k<-4$

QUESTION/*VRAAG* 6

6.1	$(0\,;-3)\quad -3=p^0+q \qquad \text{OR} \qquad y=p^x-4$ $q=-4$ $(3\,;4)\quad 4=p^3-4$ $p^3=8$ $p=2$ $\therefore f(x)=2^x-4$
6.2	$y>-4$ **OR/*OF*** $y\in(-4\,;\infty)$
6.3	$g(x)=mx+c$ E $(-3\,;0)$ $m=\dfrac{4-0}{3-(-3)}=\dfrac{2}{3}$ $y=\dfrac{2}{3}x+c$ $0=\dfrac{2}{3}(-3)+c \qquad \text{OR} \qquad y-0=\dfrac{2}{3}(x+3)$ $\therefore c=2$ $y=\dfrac{2}{3}x+2 \qquad\qquad y=\dfrac{2}{3}x+2$ **OR/*OF*** For g^{-1}: $y=mx-3$ $3=m(4)-3$ $m=\dfrac{3}{2}$ $\therefore g^{-1}(x)=\dfrac{3}{2}x-3$ For g: $2x+6=3y$ $\dfrac{2}{3}x+2=y$ $\therefore y=\dfrac{2}{3}x+2$

6.4	$g(x)=\frac{2}{3}x+2$ $x=\frac{2}{3}y+2$ $g^{-1}(x)=\frac{3}{2}x-3$

November 2024

4.1	$f(x)=a^x-1$ $-\frac{5}{9}=a^2-1$ $a^2=\frac{4}{9}$ $a=\frac{2}{3}$
4.2	$y>-1$ **OR/*OF*** $y\in(-1;\infty)$
4.3	

4.4	$\frac{19}{8}=\left(\frac{2}{3}\right)^{x}-1$ $\frac{27}{8}=\left(\frac{2}{3}\right)^{x}$ $\left(\frac{3}{2}\right)^{3}=\left(\frac{3}{2}\right)^{-x}$ $\therefore x=-3$ $\therefore \mathrm{C}\left(-3;\frac{19}{8}\right)$ $\mathrm{C}'\left(\frac{19}{8};-3\right)$

QUESTION/*VRAAG* 5

5.1	$p=-1$
5.2	$y=x-3$ $y=1-3$ $y=-2$ Answer only: $\frac{2}{2}$
5.3	$1=\frac{a}{0-1}-2$ $3=\frac{a}{-1}$ $a=-3$
5.4	$\frac{-3}{x-1}-2=0$ $-3=2x-2$ $2x=-1$ $\therefore x=-\frac{1}{2}$ $x\in\left[-\frac{1}{2};1\right)$ **OR/*OF*** $-\frac{1}{2}\leq x<1$

QUESTION/*VRAAG 6*

6.1	$x = \frac{-4}{2(-1)} = 2$ **OR/*OF*** $f'(x) = -2x + 4 = 0$ $x = 2$ $y = -(2)^2 + 4(2) + 5$ $= 9$ $\therefore B(2\ ;\ 9)$
6.2	$x^2 - 4x - 5 = 0$ $(x - 5)(x + 1) = 0$ $x = 5$ or $x = -1$ $\therefore A(-1\ ;\ 0)$ Equation of AC: $\frac{8-0}{3+1} = \frac{y-0}{x+1}$ **OR/*OF*** $m = \frac{8-0}{3+1} = 2$ $8x + 8 = 4y$ $y - 0 = 2(x + 1)$ $2x + 2 = y$ $y = 2x + 2$

6.3

$EH = -x^2 + 4x + 5 - (2x + 2)$

$EH = -x^2 + 2x + 3$

Max EH at:

$-2x + 2 = 0$ **OR/*OF*** $x = \frac{-b}{2a} = \frac{-(-2)}{2(-1)}$

$x = 1$ $x = 1$

Max length of EH $= -(1)^2 + 2(1) + 3$

$= 4$

Answer only: $\frac{1}{4}$

6.4

$k(x) = f(x + m)$

$= -x^2 - 2mx - m^2 + 4x + 4m + 5$

$k'(x) = -2x - 2m + 4$

$k'(x) = 2$

$-2x - 2m + 4 = 2$

$-2x = 2m - 2$

$x = 1 - m$

Answer only: $\frac{0}{5}$

$k(1 - m) = g(1 - m)$

$-(1 - m)^2 - 2m(1 - m) - m^2 + 4(1 - m) + 4m + 5 = 2(1 - m) + 2$

$-1 + 2m - m^2 - 2m + 2m^2 - m^2 + 4 - 4m + 4m + 5 = 2 - 2m + 2$

$-2m - 4 = 0$

$\therefore m = -2$

OR/*OF*

$k(x) = f(x + m) = g(x)$

$-x^2 - 2mx - m^2 + 4x + 4m + 5 = 2x + 2$

$0 = x^2 + (2m - 2)x + m^2 - 4m - 3$

$\Delta = (2m - 2)^2 - 4(1)(m^2 - 4m - 3)$

$\Delta = 4m^2 - 8m + 4 - 4m^2 + 16m + 12$

$\Delta = 8m + 16 = 0$

$\therefore m = -2$

OR/*OF*

$f(x) = -x^2 + 4x + 5$

$f'(x) = -2x + 4 = 2$

$-2x = -2$

$x = 1$

$f(1) = -(1)^2 + 4(1) + 5$

$\therefore y = 8$

$\therefore$ translate 2 units to the right

$\therefore m = -2$