DIFFERENTIAL CALCULUS

2026 WORKSHEET

June 2025

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = x^2 - 2$ (5)

8.2 Determine:

8.2.1 $\frac{d}{dx}\left[3x^2 - 4x\right]$ (2)

8.2.2 $g'(x)$ if $g(x) = -2\sqrt{x}(x-1)^2$ (4)

8.3 Given that $y = 4x - 14$ is a common tangent to $f(x) = 2x^2 - 4x - 6$ and $g(x) = ax^2 + bx - 18$, calculate the values of a and b. (6)
[17]

QUESTION 9

Given: $f(x) = x^3 - 6x^2 + 9x - 4 = (x-4)(x-k)^2$

9.1 Show that $k = 1$. (2)

9.2 Calculate the coordinates of the turning points of f. (4)

9.3 Describe the concavity of f at $x = -3$. (2)

9.4 Draw the graph of f. Label ALL turning points and intercepts with the axes. (4)

9.5 Calculate the maximum vertical distance between f and h for $1 < x < 3$, if $h(x) = -2f'(x)$. (6)
[18]

November 2024

QUESTION 8

8.1 Determine:

8.1.1 $\frac{d}{dx}\left[3x-5x^2\right]$ (2)

8.1.2 $g'(x)$ if $g(x)=\frac{2}{x^2}-\sqrt[3]{x^7}$ (4)

8.2 Determine the equation of the tangent to $f(x)=x^3-4x^2+2x+3$ at $x=2$. (3)

8.3 Given: $f(x)=-6x^2$

8.3.1 Determine $f'(x)$ from first principles. (5)

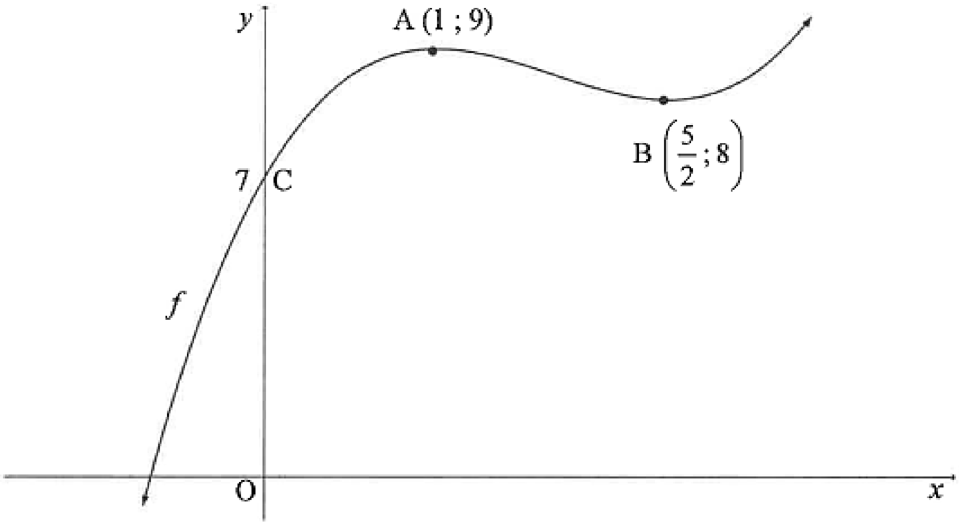
8.3.2 Write down how you will restrict the domain of f such that f^{-1}, the inverse of f, is a function. (1)

8.3.3 Determine the equation of f^{-1} for $f^{-1}(x)\leq 0$. Write your answer in the form $y=\ldots$ (3)

[18]

QUESTION 9

A(1 ; 9) and $B\left(\frac{5}{2};8\right)$ are the turning points of graph f below.
C(0 ; 7) is the y-intercept of f.



9.1 For which values of x is f decreasing? (2)

9.2 Write down the x-intercepts of f', the derivative of f. (2)

9.3 For which values of x will f be concave up? (2)

9.4 Determine the value of k for which $y = f(x) + k$ will have THREE positive x-intercepts. (2)
[8]

QUESTION 10

A cyclist rode from town P and stopped at town T. The speed (in km/h) at which this cyclist rode, is represented by the equation $s'(t) = -3t^2 + 18t$.

NOTE: Speed is the rate of change in distance with respect to time.

10.1 Calculate the maximum speed that the cyclist reached on this ride. (3)

10.2 Calculate the distance between town P and town T. (5)
[8]

WORKSHEET SOLUTIONS

June 2025

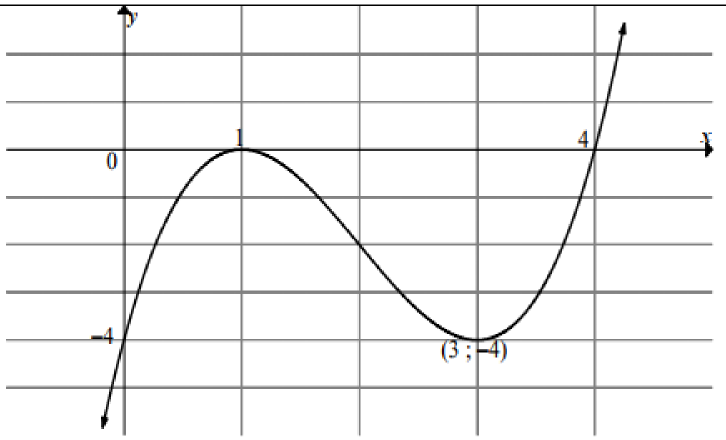
QUESTION/*VRAAG* 8

8.1	$f(x) = x^2 - 2$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2 - (x^2 - 2)}{h}$ $f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$ $f'(x) = \lim_{h \to 0} \frac{h(2x+h)}{h}$ $f'(x) = \lim_{h \to 0} (2x+h)$ $\therefore f'(x) = 2x$ **OR/*OF*** $f(x) = x^2 - 2$ $f(x+h) = (x+h)^2 - 2 = x^2 + 2xh + h^2 - 2$ $f(x+h) - f(x) = 2xh + h^2$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$ $f'(x) = \lim_{h \to 0} \frac{h(2x+h)}{h}$ $f'(x) = \lim_{h \to 0} (2x+h)$ $\therefore f'(x) = 2x$

8.2.1	$\frac{d}{dx}\left[3x^2 - 4x\right]$ $= 6x - 4$
8.2.2	$g(x) = -2\sqrt{x}(x-1)^2$ $g(x) = -2x^{\frac{1}{2}}(x^2 - 2x + 1)$ $g(x) = -2x^{\frac{5}{2}} + 4x^{\frac{3}{2}} - 2x^{\frac{1}{2}}$ $g'(x) = -5x^{\frac{3}{2}} + 6x^{\frac{1}{2}} - x^{\frac{-1}{2}}$

8.3	$y = 4x - 14$ tangent $\therefore 4x - 4 = 4$ $4x = 8$ $x = 2$ $\therefore y = 4(2) - 14$ at point $(2\ ; -6)$ $g(2) = a(2)^2 + b(2) - 18 = -6$ $4a + 2b = 12 \quad ...(1)$ $g'(2) = 2a(2) + b = 4$ $4a + b = 4 \quad ...(2)$ $(1) - (2)$ $b = 8$ $4a + b = 4$ $4a + 8 = 4$ $4a = -4$ $a = -1$
	OR/OF $2x^2 - 4x - 6 = 4x - 14$ $2x^2 - 8x + 8 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $\therefore x = 2$ $\therefore y = 4(2) - 14$ at point $(2\ ; -6)$ $g(2) = a(2)^2 + b(2) - 18 = -6$ $4a + 2b = 12 \quad ...(1)$ $g'(2) = 2a(2) + b = 4$ $4a + b = 4 \quad ...(2)$ $(1) - (2) \quad b = 8$ $4a + b = 4$ $4a + 8 = 4$ $4a = -4$ $a = -1$

QUESTION/*VRAAG* 9

9.1	$f(x)=(x-4)(x^2-2x+1)$ $f(x)=(x-4)(x-1)^2$ $\therefore k=1$ **OR/*OF*** $f(1)=1-6+9-4=0$ $(x-1)$ is a factor $\therefore k=1$ **OR/*OF*** $-4k^2=-4$ $k^2=1$ $\therefore k=1$
9.2	$3x^2-12x+9=0$ $x^2-4x+3=0$ $(x-3)(x-1)=0$ $x=3$ or $x=1$ TP's: $(3\,;-4)$ and $(1\,;0)$
9.3	$f''(x)=6x-12$ $f''(-3)=-30$ $f''(x)<0$, therefore concave down
9.4	
9.5	$\text{Distance}=(-6x^2+24x-18)-(x^3-6x^2+9x-4)$ $=-6x^2+24x-18-x^3+6x^2-9x+4$ $=-x^3+15x-14$ Max distance : $-3x^2+15=0$ $3x^2=15$ $x^2=5$ $x=\pm\sqrt{5}$, but $1<x<3$ $\therefore x=\sqrt{5}$ $d(\sqrt{5})=8{,}36$ max distance = 8,36

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QUESTION/*VRAAG* 8

8.1.1	$\frac{d}{dx}\left[3x-5x^2\right]$ $=3-10x$
8.1.2	$g(x)=\frac{2}{x^2}-\sqrt[3]{x^7}$ $g(x)=2x^{-2}-x^{\frac{7}{3}}$ $g'(x)=-4x^{-3}-\frac{7}{3}x^{\frac{4}{3}}$
8.2	$f(x)=x^3-4x^2+2x+3$ $f'(x)=3x^2-8x+2$ $m=f'(2)=3(2)^2-8(2)+2$ $m=-2$ $y=f(2)=(2)^3-4(2)^2+2(2)+3$ $y=-1$ $y+1=-2(x-2)$ $y=-2x+3$
8.3.1	$f(x)=-6x^2$ $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to0}\frac{-6(x+h)^2+6x^2}{h}$ $=\lim_{h\to0}\frac{-6x^2-12xh-6h^2+6x^2}{h}$ $=\lim_{h\to0}\frac{-12xh-6h^2}{h}$ $=\lim_{h\to0}\frac{h(-12x-6h)}{h}$ $=\lim_{h\to0}(-12x-6h)$ $=-12x$

	OR/*OF* $f(x) = -6x^2$ $f(x+h) = -6x^2 - 12xh - 6h^2$ $f(x+h) - f(x) = -6x^2 - 12xh - 6h^2 + 6x^2 = -12xh - 6h^2$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{-12xh - 6h^2}{h}$ $= \lim_{h \to 0} \frac{h(-12x - 6h)}{h}$ $= \lim_{h \to 0} (-12x - 6h)$ $= -12x$
8.3.2	$x \geq 0$ **OR/*OF*** $x \leq 0$
8.3.3	$y = -6x^2$ $x = -6y^2$ $y^2 = \frac{-1}{6}x$ $y = \pm\sqrt{-\frac{1}{6}x}$ $\therefore y = -\sqrt{-\frac{1}{6}x} \quad ; \quad x \leq 0$

QUESTION/*VRAAG* 9

9.1	$1 < x < \frac{5}{2}$
9.2	$(1\,;0)$ and $\left(\frac{5}{2};0\right)$
9.3	$\dfrac{\frac{5}{2}+1}{2}$ $=\frac{7}{4}$ Concave up for $x > \frac{7}{4}$
9.4	$-9 < k < -8$

QUESTION/*VRAAG* 10

10.1	$-6t+18=0$ $18=6t$ $3=t$ $s'(3)=-3(3)^2+18(3)=27$
10.2	$-3t^2+18t=0$ $-3t(t-6)=0$ $t=0$ or $t=6$ $s(t)=at^3+bt^2$ $s'(t)=3at^2+2bt$ $3a=-3$ and $2b=18$ $a=-1$ $b=9$ $s(t)=-1t^3+9t^2$ $s(6)=-(6)^3+9(6)^2$ $s(6)=108$