EUCLIDEAN GEOMETRY

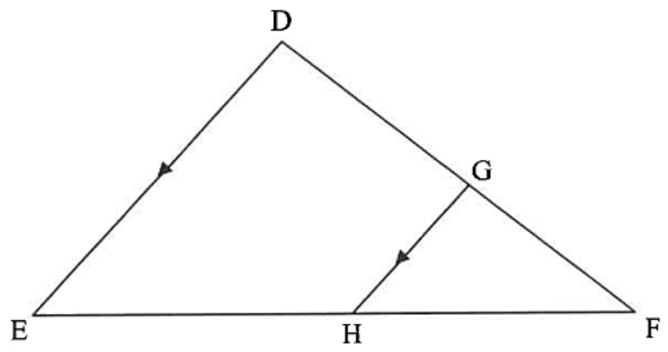
2026 WORKSHEET

June 2025

Provide reasons for your statements in QUESTIONS 9 and 10.

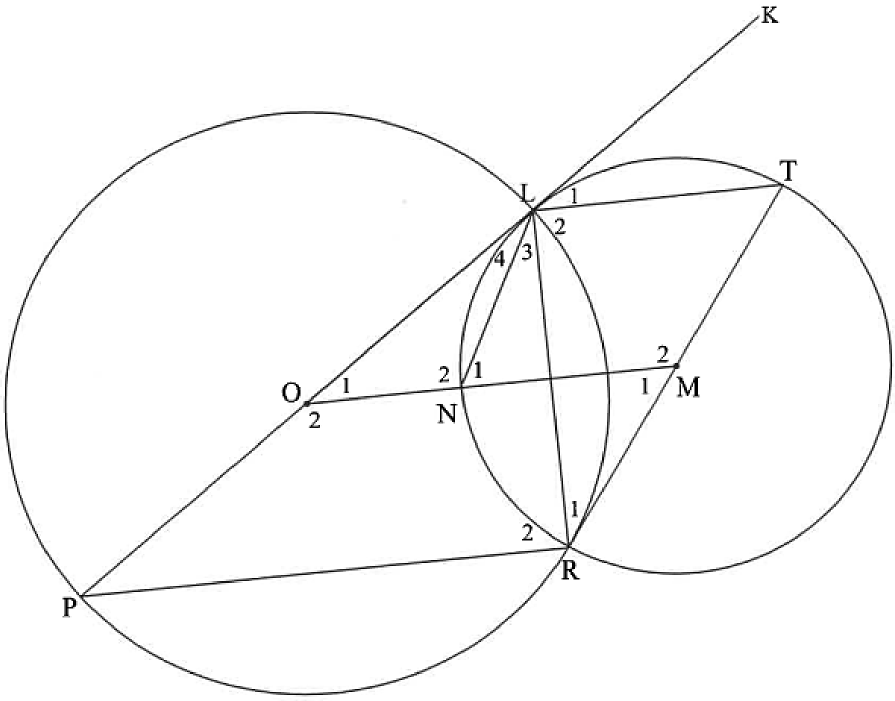
QUESTION 9

9.1 In the diagram, ΔDEF is drawn. Line GH intersects DF and EF at G and H respectively such that $GH \parallel DE$ and $\frac{GF}{DG} = \frac{2}{5}$.



9.1.1	Write down, with a reason, the value of $\frac{HF}{EH}$.	(2)
9.1.2	If $EF = 21$ cm, calculate the length of EH.	(2)
9.1.3	Write down a triangle which is similar to ΔFGH.	(1)
9.1.4	Hence, calculate the value of $\frac{GH}{DE}$.	(2)

9.2 In the diagram, POL is a diameter of the larger circle with centre O. TMR is a diameter of the smaller circle with centre M. The two circles intersect at L and R. PLK is a tangent to the smaller circle at L and TR is a tangent to the larger circle at R. OM intersects the smaller circle at N. Straight lines LT, LR, LN and PR are drawn.



Prove, giving reasons, that:

9.2.1 LT || PR (4)

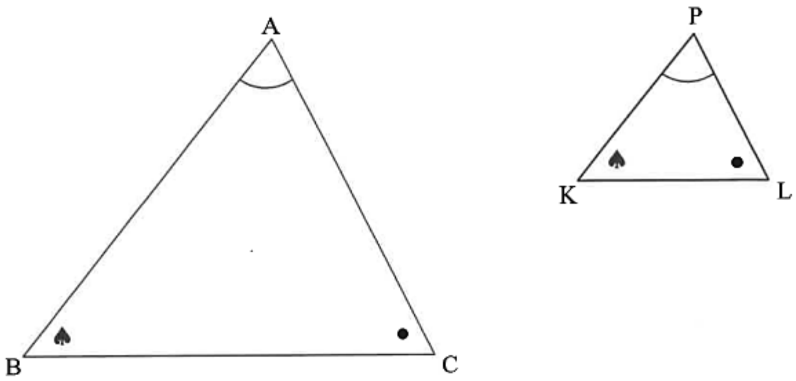
9.2.2 LORM is a cyclic quadrilateral, if it is also given that LT || OM (5)

9.2.3 LN bisects $O\hat{L}R$ (4)

[20]

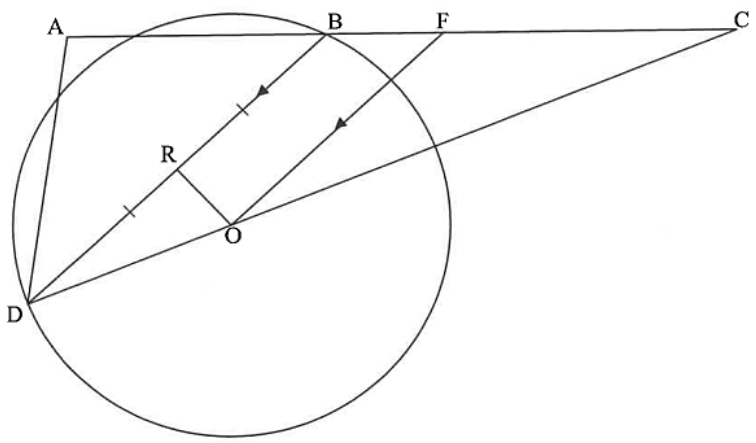
QUESTION 10

10.1 In the diagram, ΔABC and ΔPKL are drawn such that $\hat{A}=\hat{P}$, $\hat{B}=\hat{K}$ and $\hat{C}=\hat{L}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. that $\frac{AB}{PK}=\frac{AC}{PL}$. (6)

10.2 In the diagram, O is the centre of the circle. Points D and B lie on the circle. Points A and C lie outside the circle such that side AC of ΔADC passes through B. F is a point on BC such that $FO \parallel BD$. $DR = RB$ and RO is drawn.



10.2.1 Prove, with reasons, that $\Delta CFO \,|||\, \Delta CBD$. (3)

10.2.2 If it is given that $R\hat{D}O = F\hat{C}O$, show, with reasons, that $OF.CD = CO.BC$ (2)

10.2.3 It is further given that $DC = 19{,}2$ units, $BD = 12$ units and $\frac{RO}{RD}=\frac{3}{4}$

Prove, with reasons, that $BF=\frac{75}{16}$ (6)

10.2.4 Calculate the size of $A\hat{B}D$. (3)

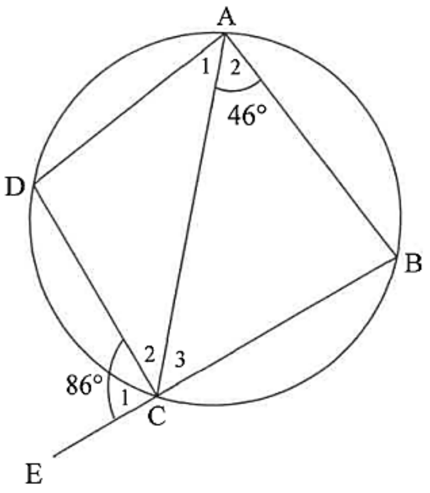
[20]

November 2024

Provide reasons for your statements in QUESTIONS 9, 10 and 11.

QUESTION 9

In the diagram, ABCD is a cyclic quadrilateral. BC is produced to E. AC is drawn.
$\hat{A}_1 = \frac{1}{2}\hat{B}$, $\hat{A}_2 = 46°$ and $\hat{C}_1 = 86°$.

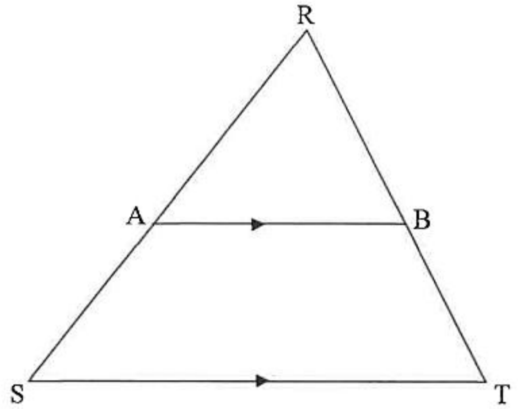


9.1 Calculate, with a reason, the value of $\hat{A}_1$. (2)

9.2 Hence, prove that AD = DC. (4)
[6]

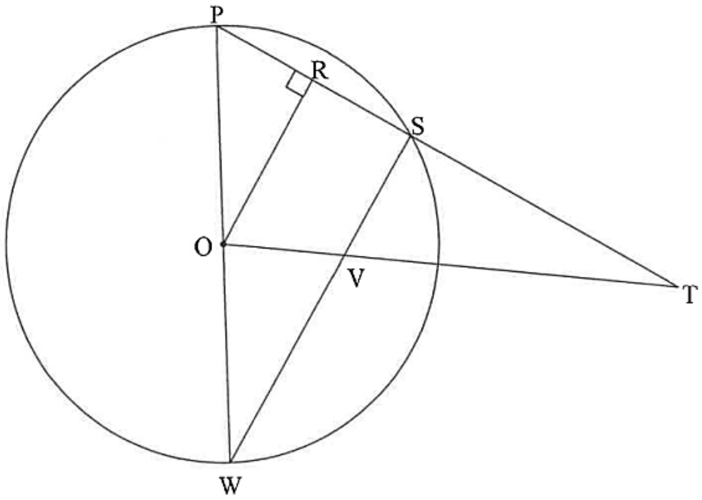
QUESTION 10

10.1 In the diagram, ΔRST is drawn. Line AB intersects RS and RT at A and B respectively such that AB || ST.



Prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. $\frac{RA}{AS} = \frac{RB}{BT}$ (6)

10.2 In the diagram, O is the centre of the circle. ΔPWS is drawn with P, W and S on the circle. OR $\perp$ PS. PRS is produced to T. SW and OT intersect at V. OV : OT = 1 : 4

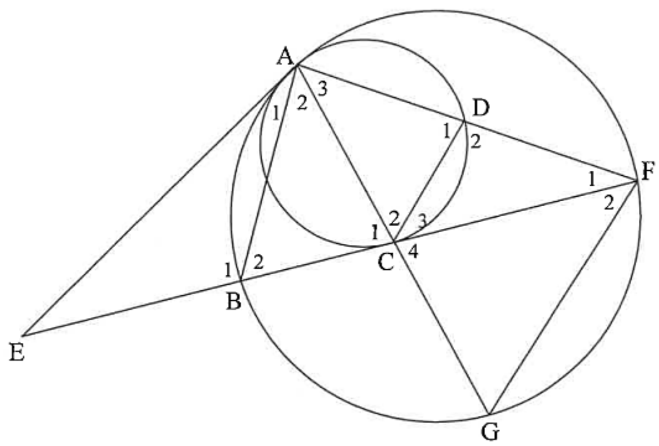


10.2.1 Prove, with reasons, that OR : WS = 1 : 2 (5)

10.2.2 Calculate the length of PT if ST = 15 units. (4)
[15]

QUESTION 11

In the diagram, A, B, G and F lie on the larger circle. A smaller circle is drawn to touch the larger circle internally at A. EA is a common tangent to both circles. EBCF is a tangent to the smaller circle at C. AC is produced to G. AF cuts the smaller circle at D. AB, CD and GF are drawn.



11.1 If $\hat{EAG} = x$, determine with reasons, FOUR other angles that are equal to x. (6)

11.2 Prove that AG.AD = AC.AF (4)

11.3 Prove that ΔAGF ||| ΔABC (4)

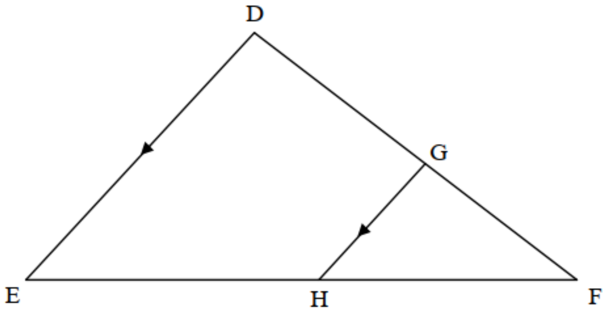
11.4 Prove that $GF^2 = \dfrac{BC.FC.AF}{AD}$ (6)
[20]

WORKSHEET SOLUTIONS

June 2025

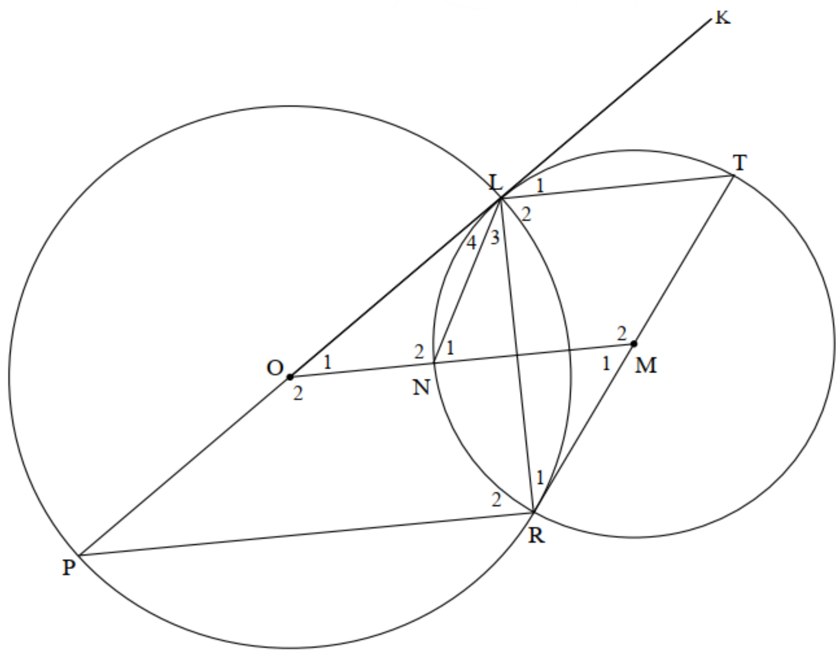
QUESTION/*VRAAG* 9

9.1



9.1.1	$\frac{HF}{EH}=\frac{GF}{GD}=\frac{2}{5}$ [line || to one side of Δ/*lyn || een sy van* Δ] **OR** [prop theorem; GH||DE/*eweredigheidst.; GH || DE*]
9.1.2	$\frac{EH}{EF}=\frac{DG}{DF}=\frac{5}{7}$ [line || to one side of Δ/*lyn || een sy van* Δ] **OR** [prop theorem; GH||DE/*eweredigheidst.; GH || DE*] $\frac{EH}{21}=\frac{5}{7}$ $EH=15cm$ **OR/*OF*** $\frac{HF}{EF}=\frac{2}{7}$ [line || to one side of Δ/*lyn || een sy van* Δ] **OR** [prop theorem; GH||DE/*eweredigheidst.; GH || DE*] $\frac{HF}{21}=\frac{2}{7}$ $HF=6cm$ $EH=21-6$ $EH=15cm$
9.1.3	ΔFGH ||| **ΔFDE** [∠∠∠]
9.1.4	$\frac{GH}{DE}=\frac{FH}{FE}$ [|||Δ's] **OR/*OF*** $\frac{GH}{DE}=\frac{FG}{FD}$ [|||Δ's] $\frac{GH}{DE}=\frac{2}{7}$

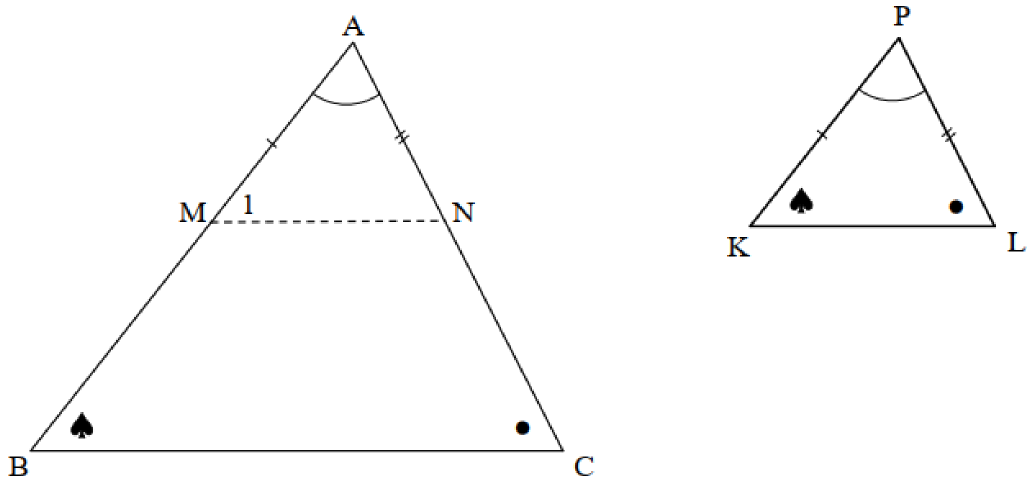
9.2



9.2.1	$\hat{L}_2 = 90°$ [$\angle$ in semi-circle/ *$\angle$ in halwe sirkel*] $\hat{R}_2 = 90°$ [$\angle$ in semi-circle] *$\angle$ in halwe sirkel*] $\therefore \hat{L}_2 = \hat{R}_2$ $\therefore LT \parallel PR$ [alt $\angle$s =/*verw.* $\angle$*e* =] **OR** $\hat{L}_1 = \hat{R}_1$ [tan chord theorem/ *raaklyn-koordst.*] $\hat{R}_1 = \hat{P}$ [tan chord theorem] *raaklyn-koordst.*] $\therefore \hat{L}_1 = \hat{P}$ $\therefore LT \parallel PR$ [corresp. $\angle$s =/ *ooreenk.* $\angle$*e* =]
9.2.2	$\hat{L}_1 = \hat{R}_1$ [tan chord theorem/*raaklyn-koordst.*] $\hat{L}_1 = \hat{O}_1$ [corresp. $\angle$s; LT ‖ OM/*ooreenk.* $\angle$*e; LT* ‖*OM*] $\therefore \hat{R}_1 = \hat{O}_1$ $\therefore$ L, O, R and M are concyclic. $\therefore$ LORM is a cyclic quadrilateral [converse $\angle$s in the same seg/ *omgekeerde* $\angle$*e in dies. sirkel segm*]
9.2.3	$O\hat{L}R = \hat{M}_1$ [$\angle$s in the same seg/$\angle$*e in dieselfde segment*] $2\hat{L}_3 = \hat{M}_1$ [$\angle$ at centre = 2 × $\angle$ at circumference/ *midpt.* $\angle$ = *2*× *omtreks* $\angle$] $\therefore O\hat{L}R = 2\hat{L}_3$ $\therefore \hat{L}_4 = \hat{L}_3$ $\therefore$ LN bisects $O\hat{L}R$

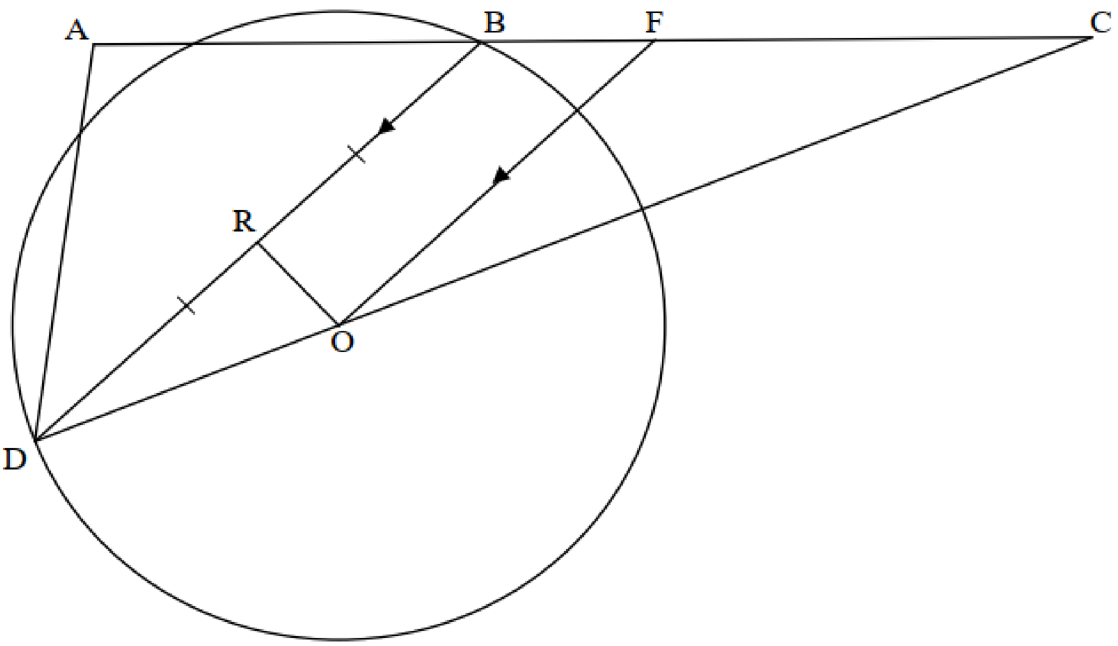
QUESTION/*VRAAG* 10

10.1



10.1	Construction: Draw line MN, where M and N are points on AB and AC respectively such that AM = PK and AN = PL. In Δ AMN and ΔPKL $\hat{A} = \hat{P}$ [given] AM = PK [construction] AN = PL [construction] ΔAMN ≡ ΔPKL [s∠s] $\therefore \hat{M}_1 = \hat{K}$ But $\hat{B} = \hat{K}$ [given] $\therefore \hat{M}_1 = \hat{B}$ ∴ MN ∥ BC [corresp ∠s =/*ooreenk. ∠e* =] $\therefore \frac{AB}{AM} = \frac{AC}{AN}$ [line ∥ one side of Δ/*lyn* ∥ *een sy v* Δ] But AM = PK and AN = PL $\therefore \frac{AB}{PK} = \frac{AC}{PL}$

10.2



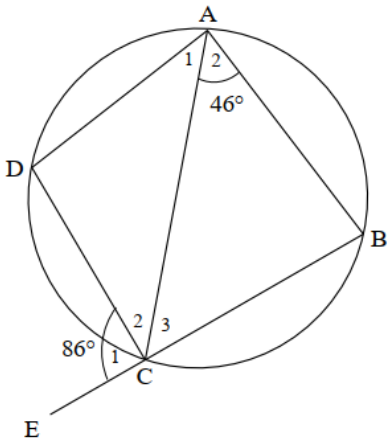
10.2.1	In ΔCFO and ΔCBD $\hat{C} = \hat{C}$ [common] $C\hat{F}O = C\hat{B}D$ [corresp $\angle$s; BD‖FO/*ooreenk. $\angle$e; BD‖FO*] $C\hat{O}F = C\hat{D}B$ [sum of $\angle$s in Δ/*binne $\angle$e v Δ*] **OR** [corresp $\angle$s; BD‖FO/ *ooreenk. $\angle$e; BD‖FO*] ΔCFO ‖‖ ΔCBD [$\angle\angle\angle$]
10.2.2	$\frac{FO}{BD} = \frac{CO}{CD}$ [‖‖ Δs] FO.CD = CO.BD But $R\hat{D}O = F\hat{C}O$ [given] $\therefore$ BD = BC [sides opp equal $\angle$s/*sye teenoor gelyke $\angle$e*] $\therefore$ FO.CD = CO.BC

10.2.3	RD = 6 units [DR = RB] $\frac{RO}{RD}=\frac{3}{4}$ $\therefore RO=\frac{3}{4}(6)$ RO = 4,5 units OR $\perp$ BD [line from centre to midpt of chord/ *midpt. sirkel; midpt koord*] $\therefore DO=\sqrt{6^2+4,5^2}$ [Pythagoras] DO = 7,5 units $\frac{BF}{BC}=\frac{DO}{DC}$ [line || to one side of Δ/*lyn* || *een sy v* Δ] **OR/*OF*** [prop theorem; BD||FO/*eweredigheidst.*;BD||FO] BC = BD = 12 units [sides opp equal ∠s/*sye teenoor gelyke ∠e*] $\therefore \frac{BF}{12}=\frac{7,5}{19,2}$ $BF=\frac{7,5\times 12}{19,2}$ $BF=\frac{75}{16}$ units

10.2.4	$\tan R\hat{D}O = \frac{RO}{RD} = \frac{4,5}{6} = \frac{3}{4}$ $R\hat{D}O = 36,87°$ $F\hat{C}O = R\hat{D}O = 36,87°$ [given] $\therefore A\hat{B}D = 73,74°$ [ext $\angle$ of Δ/*buite* $\angle$ *v.* Δ] ***OR/OF*** $CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos D\hat{B}C$ $\cos D\hat{B}C = \frac{12^2 + 12^2 - 19,2^2}{2(12)(12)}$ $\cos D\hat{B}C = -\frac{7}{25}$ $D\hat{B}C = 106,26°$ $\therefore A\hat{B}D = 73,74°$ [$\angle$s on a straight line/$\angle$*e op 'n reguitlyn*]

November 2024

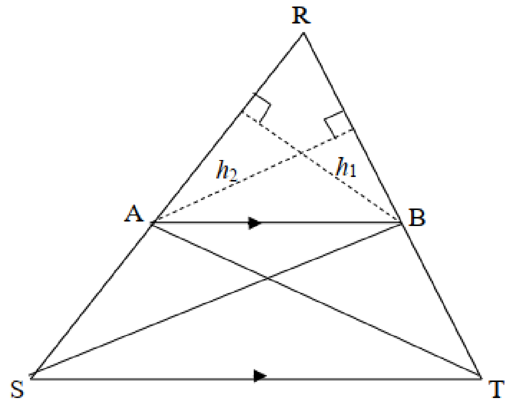
QUESTION/VRAAG 9



9.1	$\hat{A}_1 = 40°$ [ext. $\angle$ of a cyclic quad / *buite $\angle$ van kvh*]
9.2	$\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $\hat{D} = 100°$ [opp $\angle$s of cyclic quad / *teenoorst. $\angle$e van kvh*] $\therefore \hat{C}_2 = 40°$ [sum of $\angle$s in Δ / *binne $\angle$e van Δ*] $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ $\therefore$ AD = DC [sides opp = $\angle$s / *sye teenoor gelyke $\angle$*] **OR/*OF*** $\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $A\hat{C}E = \hat{A}_2 + \hat{B}$ [ext $\angle$ of Δ / *buite $\angle$ van Δ*] $\therefore \hat{C}_2 = 40°$ $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ $\therefore$ AD = DC [sides opp = $\angle$s / *sye teenoor gelyke $\angle$*] **OR/*OF*** $\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $\therefore \hat{C}_3 = 180° - 46° - 80°$ [sum of $\angle$s in Δ / *binne $\angle$e van Δ*] $\therefore \hat{C}_3 = 54°$ $\therefore \hat{C}_2 = 180° - 86° - 54°$ [$\angle$s on a str. line / *$\angle$e op 'n reguitlyn*] $\therefore \hat{C}_2 = 40°$ $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ $\therefore$ AD = DC [sides opp = $\angle$s / *sye teenoor gelyke $\angle$*]

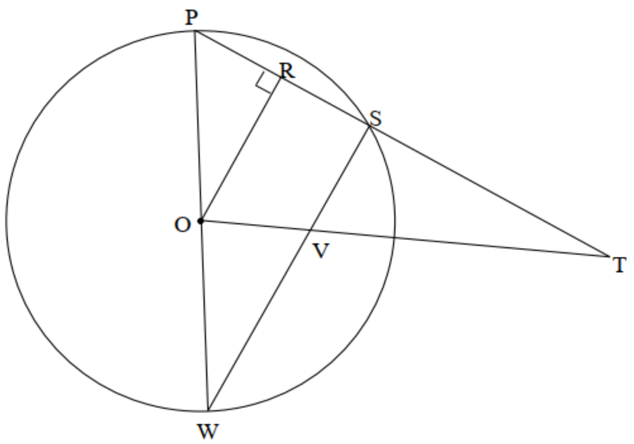
QUESTION/VRAAG 10

10.1



10.1	Construction: Join SB and TA and draw h_1 from B $\perp$ AR and h_2 from A $\perp$ RB *Konstruksie: Verbind SB en TA en trek h_1 vanaf B $\perp$ AR en h_2 vanaf A $\perp$ RB* Proof/*Bewys*: $\dfrac{\text{area } \Delta RAB}{\text{area } \Delta ASB} = \dfrac{\frac{1}{2} RA \times h_1}{\frac{1}{2} AS \times h_1} = \dfrac{RA}{AS}$ $\dfrac{\text{area } \Delta RAB}{\text{area } \Delta ABT} = \dfrac{\frac{1}{2} RB \times h_2}{\frac{1}{2} BT \times h_2} = \dfrac{RB}{BT}$ area ΔRAB = area ΔRAB [common/*gemeenskaplik*] But area ΔASB = area ΔABT [same base & height; AB || ST/ *dies. basis & hoogte; AB ||ST*] $\therefore \dfrac{\text{area } \Delta RAB}{\text{area } \Delta ASB} = \dfrac{\text{area } \Delta RAB}{\text{area } \Delta ABT}$ $\therefore \dfrac{RA}{AS} = \dfrac{RB}{BT}$

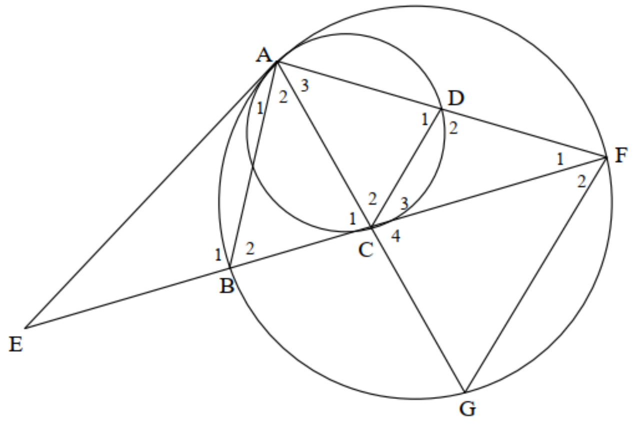
10.2



10.2.1	$PR = RS$	[line from centre $\perp$ to chord/ *lyn vanuit midpt. sirkel $\perp$ op koord*]
	$PO = OW$	[radii / *radiusse*]
	$\therefore \quad OR = \frac{1}{2}WS$	[midpt theorem/*midpt. stelling*]
	$\therefore \quad OR : WS = 1 : 2$	
	OR/*OF*	
	$P\hat{S}W = 90°$	[$\angle$ in semi circle/*$\angle$ in halwe sirkel*]
	$P\hat{R}O = 90°$	[given]
	$\therefore P\hat{R}O = P\hat{S}W$	
	$\therefore \quad RO \parallel SW$	[corresp $\angle$s = / *ooreenk. $\angle$e =*] **OR/*OF*** [co-int. $\angle$s suppl / *ko-binne $\angle$e suppl*]
	$\frac{PO}{OW} = \frac{PR}{RS}$	[prop theorem; $RO \parallel SW$/ *lyn $\parallel$ een sy van Δ*]
	$PO = OW$	[radii / *radiusse*]
	$\therefore \quad PR = RS$	
	$\therefore \quad OR : WS = 1 : 2$	[midpt theorem/ *midpt. stelling*]

	OR/*OF* ΔPRO and ΔPSW $P\hat{S}W = 90°$ [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $P\hat{R}O = 90°$ [given] $\therefore P\hat{R}O = P\hat{S}W$ $\hat{P}$ is common $P\hat{O}R = P\hat{W}S$ [sum of $\angle$s in Δ/ *som van* $\angle$*e in* Δ] $\therefore \Delta PRO \parallel\mid \Delta PSW$ [$\angle\angle\angle$] $\therefore \frac{PO}{PW} = \frac{RO}{SW}$ [$\parallel\mid \Delta$s / $\parallel\mid \Delta$*e*] but $PW = 2\,PO$ [diameter = 2 radius/*middellyn = 2 radius*] $\therefore \frac{RO}{SW} = \frac{PO}{2PO}$ $= \frac{1}{2}$ $\therefore$ OR : WS = 1 : 2
10.2.2	$\frac{OV}{VT} = \frac{RS}{ST} = \frac{1}{3}$ [prop theorem; RO $\parallel$ SW/ *lyn* $\parallel$ *een sy van* Δ] $\frac{RS}{15} = \frac{1}{3}$ RS = 5 units PR = RS = 5 units [line from centre $\perp$ to chord / *lyn vanuit midpt. sirkel* $\perp$ *op koord*] $\therefore$ PT = 25 units

QUESTION/*VRAAG* 11



11.1	$\hat{D}_1 = E\hat{A}G = x$ [tan-chord theorem/ $\angle$ *tussen raaklyn en koord*] $\hat{C}_1 = \hat{D}_1 = x$ [tan-chord theorem/ $\angle$ *tussen raaklyn en koord*] $\hat{C}_4 = \hat{C}_1 = x$ [vert opp $\angle$s = / *regoorst.* $\angle$*e*] $A\hat{F}G = E\hat{A}G = x$ [tan-chord theorem/ $\angle$ *tussen raaklyn en koord*] **OR/*OF*** $EA = EC$ [tans from common pt/ *raaklyne vanuit dies. punt*] $\hat{C}_1 = E\hat{A}G = x$ [$\angle$s opp equal sides/ $\angle$*e teenoor gelyke sye*] $\hat{C}_4 = \hat{C}_1 = x$ [vert opp $\angle$s = / *regoorst.* $\angle$*e*] $\hat{D}_1 = E\hat{A}G = x$ [tan-chord theorem/ $\angle$ *tussen raaklyn en koord*] $A\hat{F}G = E\hat{A}G = x$ [tan-chord theorem $\angle$ *tussen raaklyn en koord*]

11.2	$\hat{D}_1 = A\hat{F}G = x$	
	$\therefore$ DC $\parallel$ FG	[corresp $\angle$s = / *ooreenk* $\angle e$ =]
	$\frac{AG}{AC} = \frac{AF}{AD}$	[prop theorem; DC $\parallel$ FG / *lyn* $\parallel$ *een sy van* Δ]
	$\therefore$ AG.AD = AC.AF	
	OR/OF	
	In ΔACD and ΔAGF	
	$\hat{A}_3$ is common	
	$A\hat{F}G = \hat{D}_1 = x$	[proved in 11.1 / *reeds bewys*]
	$\hat{C}_2 = A\hat{G}F = x$	[sum $\angle$ Δs/*binne* $\angle e$ Δ]
	$\Delta ACD \parallel\!\mid \Delta AGF$	[$\angle\angle\angle$]
	$\frac{AC}{AG} = \frac{AD}{AF}$	[$\parallel\!\mid$ Δs $\therefore$ sides in proportion / $\parallel\!\mid$ Δe $\therefore$ *sye in dieselfde verhouding*]
	$\therefore$ AG.AD = AC.AF	
11.3	In ΔAGF and ΔABC	
	$\hat{G} = \hat{B}_2$	[$\angle$s in the same seg / $\angle e$ *in dies. segment*]
	$A\hat{F}G = \hat{C}_1 = x$	[proved in 11.1 / *reeds bewys*]
	$\hat{A}_3 = \hat{A}_2$	[sum of $\angle$s in Δ/*binne* $\angle e$ *van* Δ]
	$\Delta AGF \parallel\!\mid \Delta ABC$	[$\angle\angle\angle$]

11.4	$\frac{GF}{BC}=\frac{AF}{AC}$ [$\Delta AGF \parallel\!| \Delta ABC$] $\therefore GF=\frac{BC.AF}{AC}$ $\Delta ACD \parallel\!| \Delta FGC$ [$\angle\angle\angle$] $\therefore \frac{AC}{GF}=\frac{AD}{FC}$ $\therefore AC=\frac{AD.FG}{FC}$ $\therefore GF=BC.AF\div\frac{AD.FG}{FC}$ $GF=BC.AF\times\frac{FC}{AD.FG}$ $\therefore GF^2=\frac{BC.FC.AF}{AD}$
	OR/*OF* $\Delta AGF \parallel\!| \Delta ABC$ [$\angle\angle\angle$] $\frac{GF}{BC}=\frac{AF}{AC}$ $GF=\frac{AF.BC}{AC}$ $\Delta ACD \parallel\!| \Delta AGF$ [$\angle\angle\angle$] $\frac{AD}{AF}=\frac{CD}{GF}$ $GF=\frac{AF.CD}{AD}$ $GF\times GF=\frac{AF.BC}{AC}.\frac{AF.CD}{AD}$ $\Delta FCD \parallel\!| \Delta FAC$ [$\angle\angle\angle$] $\frac{FC}{FA}=\frac{CD}{AC}$ from $\parallel\!|$ Δ\`s $FC=\frac{CD.AF}{AC}$ $GF^2=\frac{AF.FC.BC}{AD}$