TRIGONOMETRY 2026 WORKSHEET

June 2025

QUESTION 5

5.1 If $\cos\theta = -\frac{5}{13}$ where $180° < \theta < 360°$, determine, **without using a calculator**, the value of:

5.1.1 $\sin^2\theta$ (3)

5.1.2 $\tan(360° - \theta)$ (2)

5.1.3 $\cos(\theta - 135°)$ (4)

5.2 Simplify the expression to a single trigonometric term: $\frac{2\cos(180° - x)\sin(-x)}{1 - 2\cos^2(90° - x)}$ (6)

5.3 Calculate the value of the following expression **without using a calculator**: $(\tan 92°)(\tan 94°)(\tan 96°) \ldots (\tan 176°)(\tan 178°)$ (4)

[19]

QUESTION 6

6.1 Prove that $2\cos^2(45° + x) = 1 - \sin 2x$. (4)

6.2 Consider the expression: $\sin(A - B) - \sin(A + B)$

6.2.1 Prove that $\sin(A - B) - \sin(A + B) = -2\cos A \sin B$. (2)

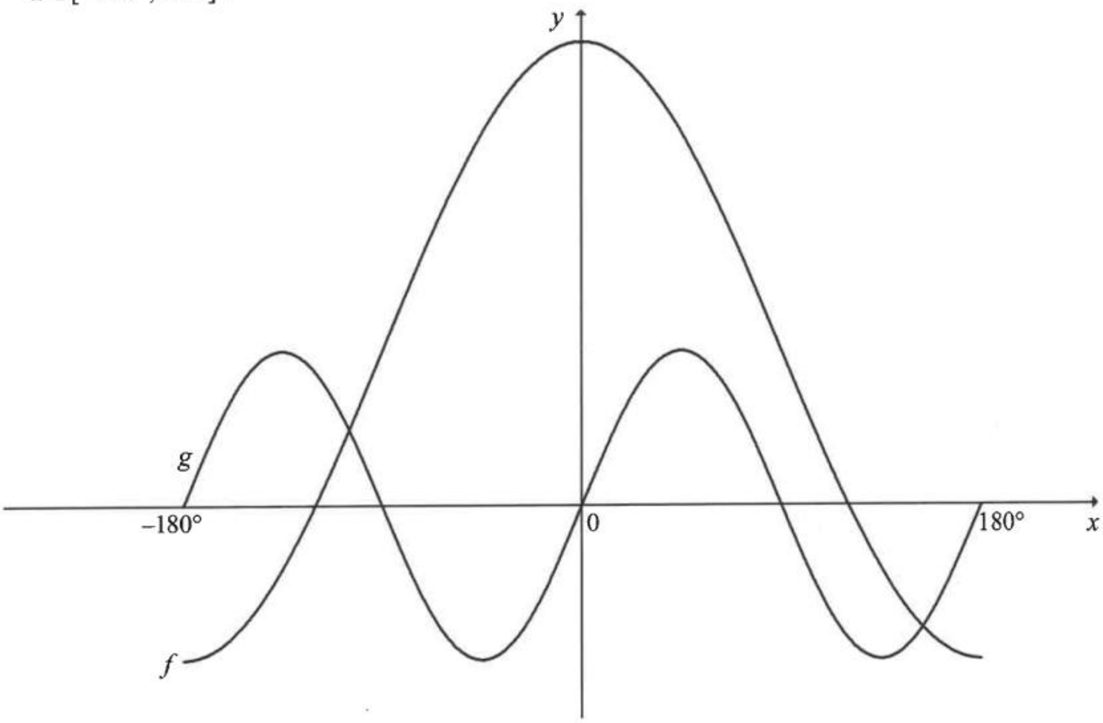
6.2.2 Simplify the following expression to a single term: $\sin 4x - \sin 10x$ (2)

6.2.3 Hence, determine the solution for $\sin 4x - \sin 10x = \sin 3x$ for $x \in [0°; 30°]$. (5)

[13]

QUESTION 7

In the diagram, the graphs of $f(x)=2\cos x+1$ and $g(x)=\sin 2x$ are drawn for the interval $x\in[-180°;180°]$.



7.1 Write down the range of f. (1)

7.2 Write down the period of g. (1)

7.3 For which values of x, in the interval $x\in[-180°;180°]$, is f increasing? (1)

7.4 Use the graphs to determine the values of x, in the interval $x\in[-180°;180°]$, for which:

7.4.1 $g(x).f'(x)<0$ (2)

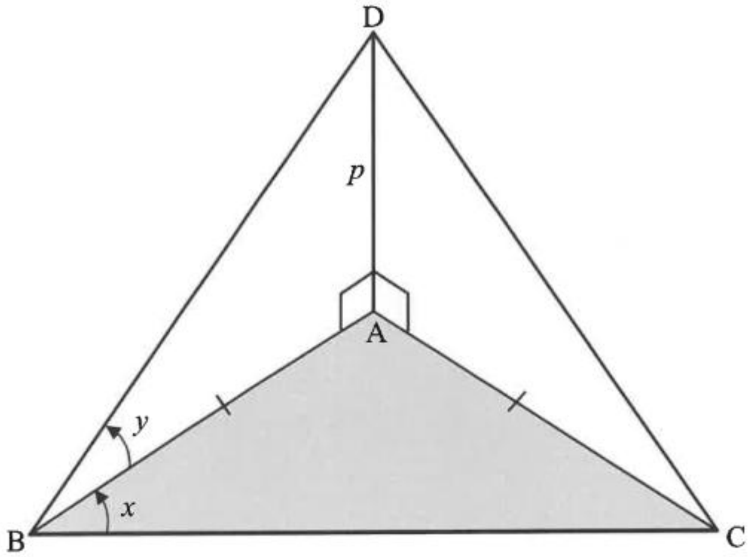
7.4.2 $\cos x\leq -\frac{1}{2}$ (3)

7.5 Graph g is shifted $45°$ to the right to obtain a new graph h. Determine the equation of h in its simplest form. (2)

[10]

QUESTION 8

In the diagram, A, B and C lie in the same horizontal plane with AB = AC. D is directly above A such that 2AD = BC. Also, AD = p, $A\hat{B}C = x$ and $D\hat{B}A = y$.



8.1 Determine AB in terms of p and y. (2)

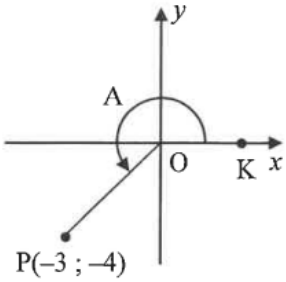
8.2 Show that $\cos x = \tan y$. (4)

8.3 If $x = 60°$, calculate the size of y. (2)
[8]

<u>**November 2024**</u>

QUESTION 5

5.1 In the diagram, line OP is given with P(–3 ; –4). $K\hat{O}P = A$.



Determine, **without using a calculator**, the value of:

5.1.1 $\cos A$ (2)

5.1.2 $\cos 2A$ (2)

5.1.3 $\sin(A-B)$, if it is further given that $\sin B = \frac{4}{5}$ and $90° < B < 360°$ (4)

5.2 If $\cos\alpha = p$, express the following expression in terms of p:

$$\frac{\cos\left(\frac{\alpha}{2} - 45°\right)\sin\left(\frac{\alpha}{2} - 45°\right)}{2}$$

(4)
[12]

QUESTION 6

6.1 Given the identity: $\cos(x-y)=\cos x\cos y+\sin x\sin y$

6.1.1 Use the compound angle identity given above to derive a formula for $\cos(x+y)$. (2)

6.1.2 Hence, or otherwise, show that:

$$\frac{\cos(90°-x)\cos y+\sin(-y)\cos(180°+x)}{\cos x\cos(360°+y)+\sin(360°-x)\sin y}=\tan(x+y)$$ (6)

6.2 Given: $f(x)=\sqrt{6\sin^2 x-11\cos(90°+x)+7}$

Solve for x in the interval $x\in(0°\,;\,360°)$ if $f(x)=2$. (6)

6.3 Consider the function: $g(x)=\dfrac{4-8\sin^2 x}{3}$

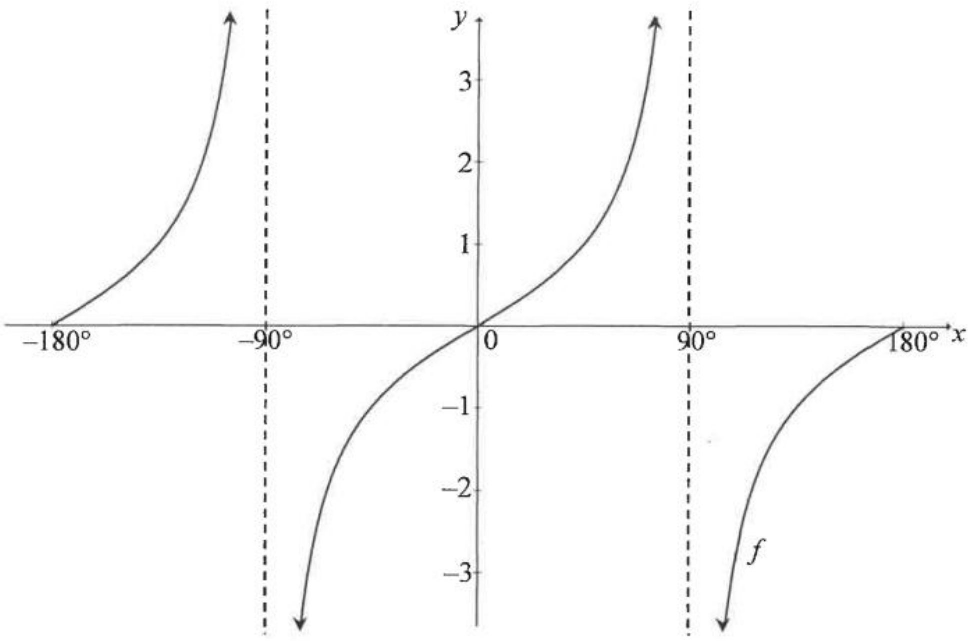
6.3.1 Calculate the maximum value of g. (3)

6.3.2 Write down the smallest possible value of x for which g will have a maximum value in the interval $x\in(0°\,;\,360°]$. (1)

[18]

QUESTION 7

In the diagram below, the graph of $f(x) = \tan x$ is drawn for the interval $x \in [-180°\,;\,180°]$.



7.1 Write down the equation of the asymptote of f in the interval $x \in [0°\,;\,180°]$. (1)

7.2 Write down the values of x in the interval $x \in [-180°\,;\,0°]$ for which $f(x) \leq 0$. (2)

7.3 Given: $g(x) = \cos 2x + 1$

7.3.1 Write down the period of g. (1)

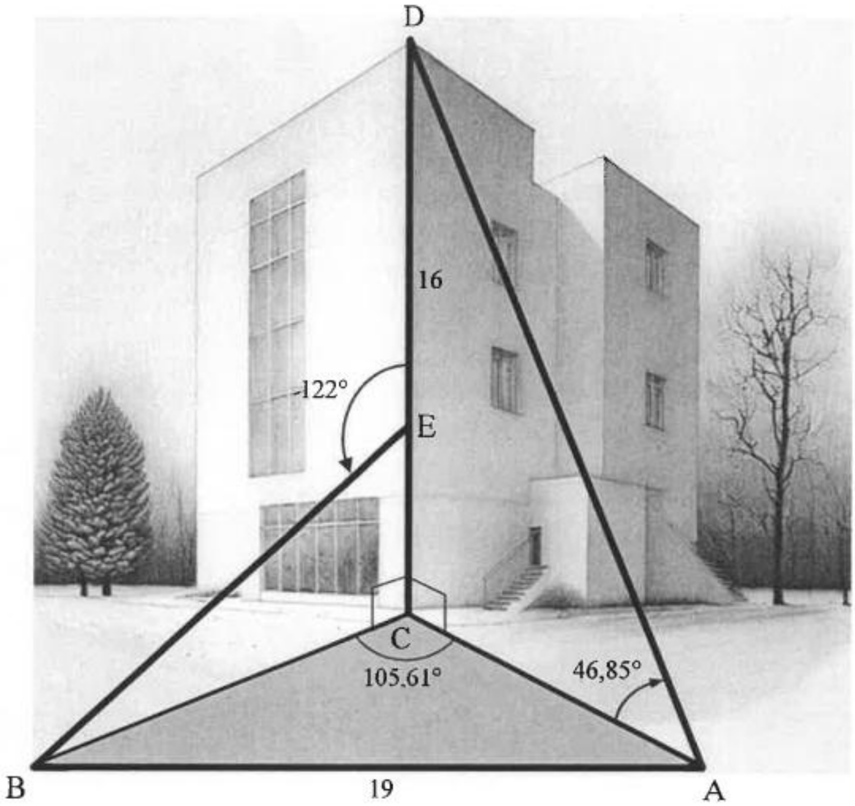
7.3.2 On the grid given in the ANSWER BOOK, draw the graph of $g(x) = \cos 2x + 1$ for the interval $x \in [-180°\,;\,180°]$. Clearly show the intercepts with the axes as well as the coordinates of the turning points. (3)

7.4 Use the graphs to determine the general solution of $2\cos^3 x - \sin x = 0$. (4)

[11]

QUESTION 8

In the diagram, C is the foot of a vertical building and D is the top of the same building. The height of the building, CD, is 16 m. Two observers are standing 19 m apart at points A and B, where A, B and C lie in the same horizontal plane. A painter is working at point E on the building. The angle of elevation of D from A is 46,85°. $D\hat{E}B = 122°$ and $B\hat{C}A = 105,61°$.



8.1 Calculate the length of AC, the distance between the observer at A and the foot of the building. (2)

8.2 Calculate how far the painter at E is from the top of the building. (7)

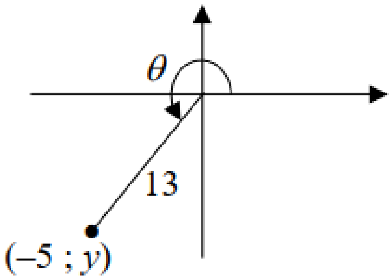
[9]

TRIGONOMETRY 2026 WORKSHEET SOLUTIONS

June 2025

QUESTION/*VRAAG* 5

5.1.1	$y^2 = \sqrt{13^2 - (-5)^2}$ [Pythagoras] $y = -12$ $\sin^2\theta$ $= \left(-\frac{12}{13}\right)^2$ $= \frac{144}{169}$ **OR/*OF*** $\sin^2\theta = 1 - \cos^2\theta$ $\sin^2\theta = 1 - \left(-\frac{5}{13}\right)^2$ $\sin^2\theta = \frac{144}{169}$
5.1.2	$\tan(360° - \theta)$ $= -\tan\theta$ $= -\left(\frac{-12}{-5}\right)$ $= -\frac{12}{5}$



5.1.3	$\cos(\theta - 135°)$ $= \cos\theta\cos 135° + \sin\theta\sin 135°$ $= \cos\theta(-\cos 45°) + \sin\theta(\sin 45°)$ $= \left(-\frac{5}{13}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{12}{13}\right)\left(\frac{\sqrt{2}}{2}\right)$ OR $\left(-\frac{5}{13}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{12}{13}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= -\frac{7\sqrt{2}}{26}$ $\quad\quad = -\frac{7}{13\sqrt{2}}$

5.2	$\frac{2\cos(180° - x)\sin(-x)}{1 - 2\cos^2(90° - x)}$ $= \frac{2(-\cos x)(-\sin x)}{1 - 2\sin^2 x}$ $= \frac{2\sin x\cos x}{\cos 2x}$ $= \frac{\sin 2x}{\cos 2x}$ $= \tan 2x$

5.3	$(\tan 92°)(\tan 94°)(\tan 96°)\ldots(\tan 176°)(\tan 178°)$ $= \left(\frac{\sin 92°}{\cos 92°}\right)\left(\frac{\sin 94°}{\cos 94°}\right)\left(\frac{\sin 96°}{\cos 96°}\right)\ldots\left(\frac{\sin 176°}{\cos 176°}\right)\left(\frac{\sin 178°}{\cos 178°}\right)$ $= \left(\frac{\cos 2°}{-\sin 2°}\right)\left(\frac{\cos 4°}{-\sin 4°}\right)\left(\frac{\cos 6°}{-\sin 6°}\right)\ldots\left(\frac{\sin 4°}{-\cos 4°}\right)\left(\frac{\sin 2°}{-\cos 2°}\right)$ $= 1$

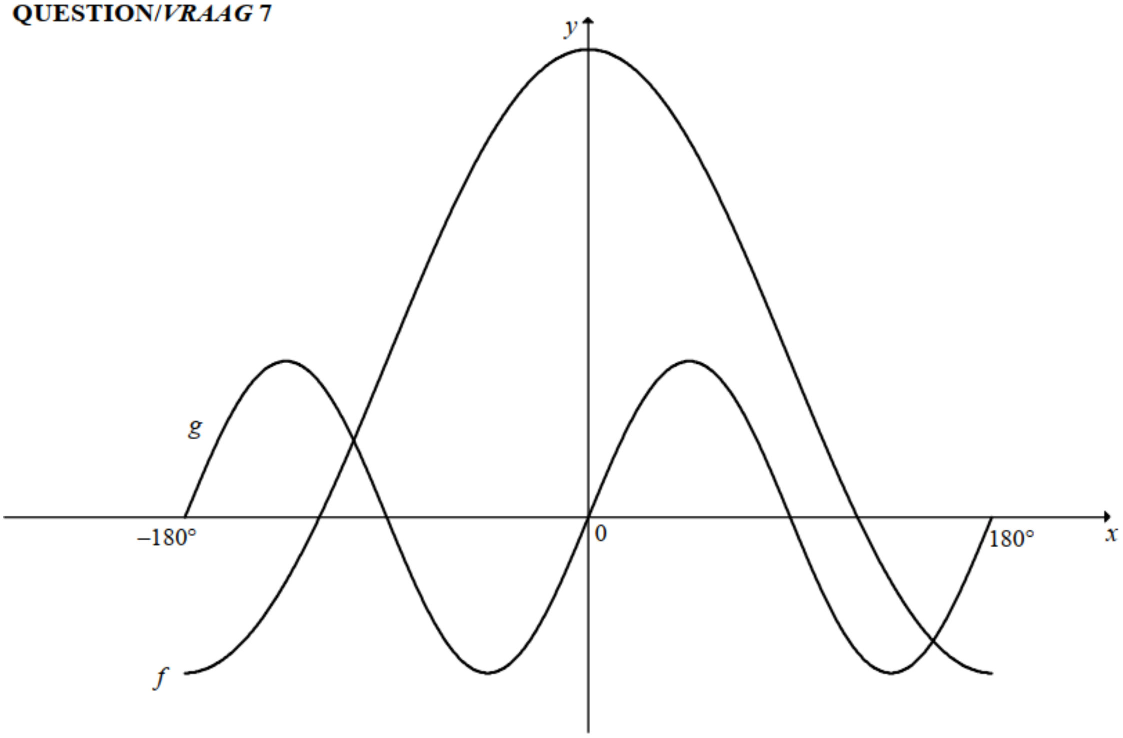
QUESTION/*VRAAG* 6

6.1	$\text{LHS} = 2\cos^2(45° + x)$ $= 2\cos^2(45° + x) + 1 - 1$ $= \cos[2(45° + x)] + 1$ $= \cos(90° + 2x) + 1$ $= (-\sin 2x) + 1$ $= 1 - \sin 2x$ $= \text{RHS}$ **OR/*OF*** $\text{LHS} = 2\cos^2(45° + x)$ $= 2(\cos(45° + x))^2$ $= 2(\cos 45° \cos x - \sin 45° \sin x)^2$ $= 2\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right)^2$ $= 2\left(\frac{1}{2}\cos^2 x - \sin x \cos x + \frac{1}{2}\sin^2 x\right)$ $= \cos^2 x - 2\sin x \cos x + \sin^2 x$ $= 1 - \sin 2x$ $= \text{RHS}$

6.2.1	$\text{LHS} = \sin(A - B) - \sin(A + B)$ $= \sin A \cos B - \cos A \sin B - (\sin A \cos B + \cos A \sin B)$ $= \sin A \cos B - \cos A \sin B - \sin A \cos B - \cos A \sin B$ $= -2\cos A \sin B$ $= \text{RHS}$
6.2.2	$\sin 4x - \sin 10x$ $= \sin(7x - 3x) - \sin(7x + 3x)$ $= -2\cos 7x \sin 3x$

6.2.3	$\sin 4x - \sin 10x = \sin 3x$ $-2\cos 7x \sin 3x = \sin 3x$ $2\cos 7x \sin 3x + \sin 3x = 0$ $\sin 3x(2\cos 7x + 1) = 0$ $\sin 3x = 0$ or $\cos 7x = -\frac{1}{2}$ $3x = 0°$; $7x = 120°$ or $7x = 240°$ $x = 0°$; $x = 17{,}14°$; $x = 34{,}29°$ N/A

QUESTION/*VRAAG* 7

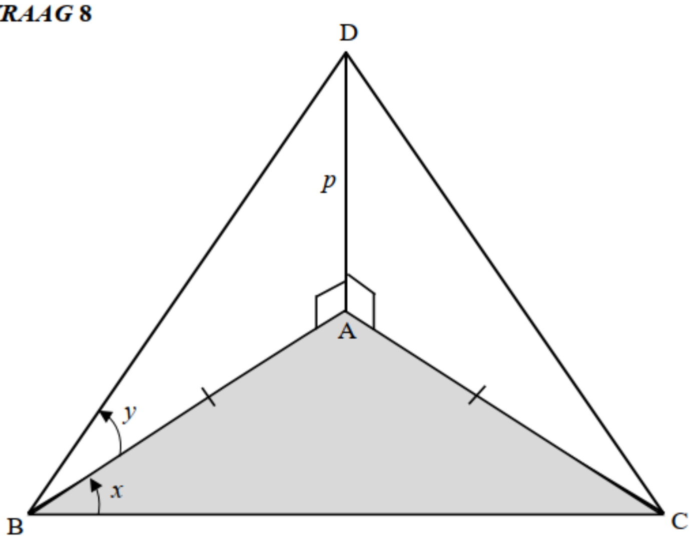


7.1	Range of *f*: $y \in [-1\,;\,3]$ **OR/*OF*** $-1 \leq y \leq 3$
7.2	Period of *g*: $180°$

7.3	f increasing: $x \in (-180° ; 0°)$ **OR/*OF*** $-180° < x < 0°$
7.4.1	$g(x).f'(x) < 0$ $x \in (-90° ; 0°) \cup (0° ; 90°)$ **OR/*OF*** $-90° < x < 0°$ or $0° < x < 90°$

7.4.2	$\cos x \leq -\frac{1}{2}$ $2\cos x + 1 \leq 0$ $x \in [-180° ; -120°] \cup [120° ; 180°]$ **OR/*OF*** $2\cos x + 1 \leq 0$ $-180° \leq x \leq -120°$ or $120° \leq x \leq 180°$
7.5	$g(x) = \sin 2x$ $h(x) = \sin 2(x - 45°)$ $= \sin(2x - 90°)$ $= -\sin(90° - 2x)$ $= -\cos 2x$

QUESTION/*VRAAG* 8



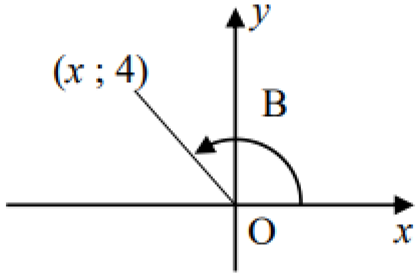
8.1	$\tan y = \dfrac{p}{AB}$ $AB = \dfrac{p}{\tan y}$	✓ correct trig ratio ✓ answer (2)
8.2	In ΔBAC: $\dfrac{\sin B\hat{A}C}{BC} = \dfrac{\sin A\hat{C}B}{AB}$ $\dfrac{\sin(180° - 2x)}{2p} = \dfrac{\sin x}{\left(\dfrac{p}{\tan y}\right)}$ $\dfrac{\sin 2x}{2p} = \sin x \times \left(\dfrac{\tan y}{p}\right)$ $\dfrac{2\sin x \cos x}{2p} = \sin x \times \left(\dfrac{\tan y}{p}\right)$ $2\cos x = \left(\dfrac{\tan y}{p}\right)(2p)$ $\cos x = \tan y$	✓ correct use of sine-rule ✓ substitute BC & AB ✓ $\sin(180° - 2x) = \sin 2x$ ✓ $\sin 2x = 2\sin x \cos x$ (4)

8.2	**OR/*OF*** In ΔBAC: $BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos B\hat{A}C$ $(2p)^2 = \left(\frac{p}{\tan y}\right)^2 + \left(\frac{p}{\tan y}\right)^2 - 2\left(\frac{p}{\tan y}\right)\left(\frac{p}{\tan y}\right)\cos(180° - 2x)$ $4p^2 = \frac{2p^2}{\tan^2 y} - \frac{2p^2(-\cos 2x)}{\tan^2 y}$ $4p^2\tan^2 y = 2p^2(1 + \cos 2x)$ $\tan^2 y = \frac{1 + 2\cos^2 x - 1}{2}$ $\tan^2 y = \cos^2 x$ $\tan y = \cos x$

8.3	$\cos x = \tan y$ $\tan y = \cos 60°$ $\tan y = 0{,}5$ $y = 26{,}57°$

November 2024

5.1.1	$r = 5$ $\cos A = -\frac{3}{5}$
5.1.2	$\cos 2A = 2\cos^2 A - 1$ $= 2\left(-\frac{3}{5}\right)^2 - 1$ $= -\frac{7}{25}$ **OR/*OF*** $\cos 2A = \cos^2 A - \sin^2 A$ $= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$ $= -\frac{7}{25}$ **OR/*OF*** $\cos 2A = 1 - 2\sin^2 A$ $= 1 - 2\left(-\frac{4}{5}\right)^2$ $= -\frac{7}{25}$

5.1.3	 $x = -3$ $\sin(A - B) = \sin A\cos B - \cos A\sin B$ $= \left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$ $= \frac{12}{25} + \frac{12}{25}$ $= \frac{24}{25}$

5.2	$\frac{\cos\left(\frac{\alpha}{2} - 45°\right)\sin\left(\frac{\alpha}{2} - 45°\right)}{2}$ $= \frac{2\cos\left(\frac{\alpha}{2} - 45°\right)\sin\left(\frac{\alpha}{2} - 45°\right)}{2.2}$ $= \frac{\sin(\alpha - 90°)}{4}$ $= \frac{-\cos\alpha}{4}$ $= \frac{-p}{4}$ **OR/*OF*** $= -\frac{1}{4}p$

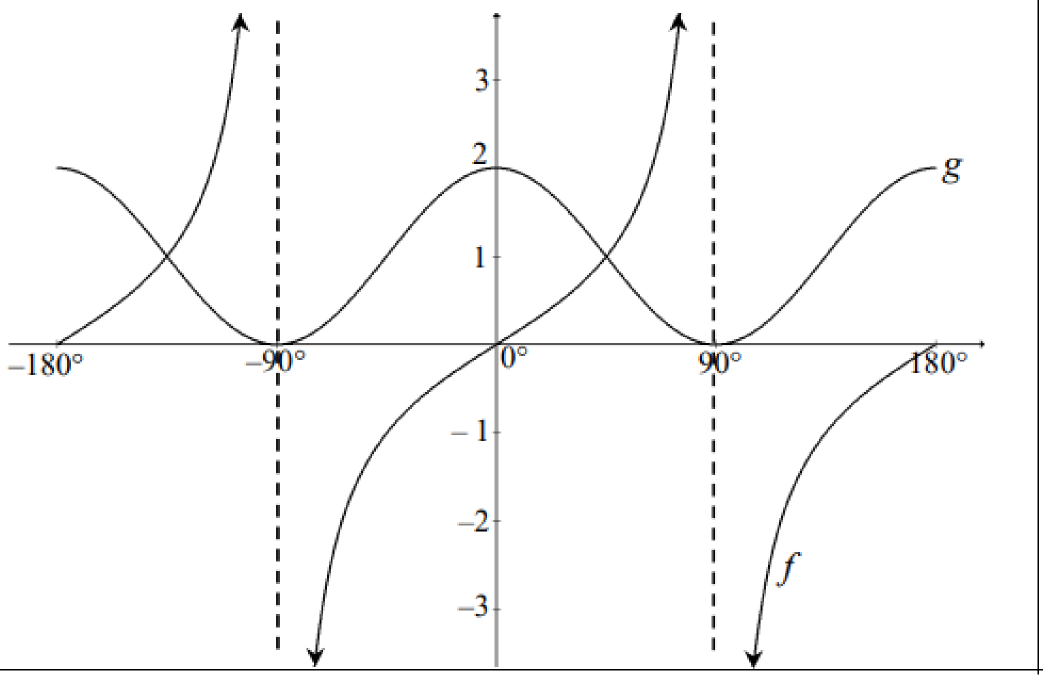
QUESTION/VRAAG 6

6.1.1	$\cos(x+y)=\cos(x-(-y))$ $=\cos x\cos(-y)+\sin x\sin(-y)$ $=\cos x\cos y-\sin x\sin y$
6.1.2	$\text{LHS}=\dfrac{\cos(90^\circ-x)\cos y+\sin(-y)\cos(180^\circ+x)}{\cos x\cos(360^\circ+y)+\sin(360^\circ-x)\sin y}$ $=\dfrac{(\sin x)\cos y+(-\sin y)(-\cos x)}{\cos x(\cos y)+(-\sin x)\sin y}$ $=\dfrac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y}$ $=\dfrac{\sin(x+y)}{\cos(x+y)}$ $=\tan(x+y)$ $=\text{RHS}$

6.2	$\sqrt{6\sin^2 x-11\cos(90^\circ+x)+7}=2$ $6\sin^2 x-11\cos(90^\circ+x)+7=4$ $6\sin^2 x-11(-\sin x)+7=4$ $6\sin^2 x+11\sin x+3=0$ $(3\sin x+1)(2\sin x+3)=0$ $\sin x=-\dfrac{1}{3}$ **OR/*OF*** $\sin x=-\dfrac{3}{2}$ ref$\angle=19{,}47^\circ$ no solution $x=199{,}47^\circ$ or $x=340{,}53^\circ$

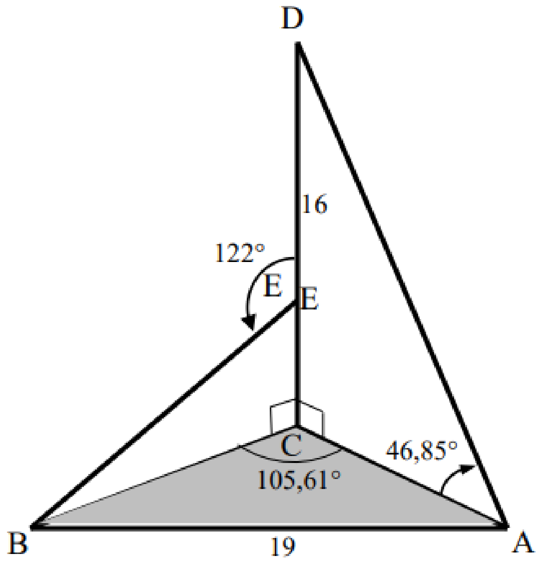
6.3.1	$g(x)=\frac{4-8\sin^2 x}{3}$ $=\frac{4(1-2\sin^2 x)}{3}$ $=\frac{4\cos 2x}{3}$ Maximum value of cos $2x$ is 1 $\therefore$ maximum value of $g(x)=\frac{4}{3}$
	OR/*OF* $4-8\sin^2 x$ is a maximum when $\sin^2 x$ is a minimum Minimum value of $\sin^2 x$ is 0 $\therefore$ max. value of $g(x)=\frac{4-8(0)}{3}$ $g(x)=\frac{4}{3}$ **OR/*OF*** $\sin x=\frac{-(0)}{2\left(-\frac{8}{3}\right)}$ $\sin x=0$ $\therefore$ max. value of $g(x)=\frac{4-8(0)}{3}$ $g(x)=\frac{4}{3}$
6.3.2	$x=180°$

QUESTION/*VRAAG* 7

7.1	$x = 90°$
7.2	$x = -180°$ or $x \in (-90° ; 0°]$ **OR/*OF*** $x = -180°$ or $-90° < x \leq 0°$
7.3.1	180°
7.3.2	

7.4	$2\cos^3 x - \sin x = 0$ $2\cos^3 x = \sin x$ $2\cos^2 x = \dfrac{\sin x}{\cos x}$ $2\cos^2 x = \tan x$ $2\cos^2 x - 1 = \tan x - 1$ $\cos 2x + 1 = \tan x$ $x = 45° + k.180°;\ k \in Z$

QUESTION/*VRAAG* 8



8.1	$\tan D\hat{A}C = \frac{DC}{AC}$ $AC = \frac{16}{\tan 46{,}85°}$ $AC = 15$ m
8.2	$(AB)^2 = (BC)^2 + (AC)^2 - 2(BC)(AC)\cos B\hat{C}A$ $(19)^2 = x^2 + (15)^2 - 2x(15)\cos 105{,}61°$ $x^2 + 8{,}07x - 136 = 0$ $x = \frac{-8{,}07 \pm \sqrt{(8{,}07)^2 - 4(1)(-136)}}{2(1)}$ $x = 8{,}30$ m or $x \neq -16{,}38$ m $B\hat{E}C = 58°$ **OR/*OF*** $E\hat{B}C = 32°$ $\tan B\hat{E}C = \frac{BC}{EC}$; $\tan E\hat{B}C = \frac{EC}{BC}$ $EC = \frac{8{,}3}{\tan 58°}$; $EC = 8{,}3 \tan 32°$ $EC = 5{,}19$ m ; $EC = 5{,}19$ m $DE = 10{,}81$ m ; $DE = 10{,}81$ m

OR/*OF*

$$\frac{\sin 105{,}61^\circ}{19} = \frac{\sin C\hat{B}A}{15}$$

$C\hat{B}A = 49{,}5^\circ$

$B\hat{A}C = 24{,}89^\circ$

$$\frac{BC}{\sin 24{,}89^\circ} = \frac{19}{\sin 105{,}61^\circ}$$

$BC = 8{,}3 \text{ m}$

$B\hat{E}C = 58^\circ$

$$\tan B\hat{E}C = \frac{BC}{EC}$$

$$EC = \frac{8{,}3}{\tan 58^\circ}$$

$EC = 5{,}19 \text{ m}$

$DE = 10{,}81 \text{ m}$

OR/*OF*

$E\hat{B}C = 32^\circ$

$$\tan E\hat{B}C = \frac{EC}{BC}$$

$EC = 8{,}3 \tan 32^\circ$

$EC = 5{,}19 \text{ m}$

$DE = 10{,}81 \text{ m}$