

**NUMBERS
BEYOND
MEASURE**



MATHEMATICS

GRADE 12

2026

**QUESTION PAPER BANK
SOLUTIONS**

PAPER 1 AND PAPER 2

$$(5) \quad 12^{5+3x} = 1$$

$$5 + 3x = 0$$

$$x = \frac{-5}{3}$$

$$(b) \quad 2x - y = 8 \dots\dots\dots ①$$

$$x^2 - xy + y^2 = 19 \dots\dots ②$$

$$① : \quad y = 2x - 8$$

$$② : \quad x^2 - x(2x - 8) + (2x - 8)^2 = 19$$

$$x^2 - 2x^2 + 8x + 4x^2 - 32x + 64 = 19$$

$$3x^2 - 24x + 45 = 0$$

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$x = 3 \quad \text{or} \quad x = 5$$

$$y = 2 \times 3 - 8 \quad \text{or} \quad y = 2 \times 5 - 8$$

$$= -2 \qquad \qquad \qquad = 2$$

$$(c) \quad f(x) = x^{10} - 2x^5 + c$$

$$f(-1) = (-1)^{10} - 2(-1)^5 + c = 0$$

$$1 + 2 + c = 0$$

$$c = -3$$

$$(d) \quad y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$\text{At } (-1; 1): m = 2(-1)$$

$$= -2$$

PAPER B

QUESTION 1

$$(a) \quad (1) \quad \frac{4x}{2} - \frac{2x+1}{3} = 5$$

$$\frac{12x - 4x - 2}{6} = \frac{30}{6} \qquad \text{OR} \qquad 12x - 2(2x + 1) = 30$$

$$8x = 32$$

$$x = 4$$

$$\begin{aligned}
 (2) \quad & (x-5)(x-6) \leq 56 \\
 & x^2 - 11x + 30 \leq 56 \\
 & x^2 - 11x - 26 \leq 0 \\
 & (x-13)(x+2) \leq 0 \\
 & \text{Critical Values: } 13 ; -2 \\
 & -2 \leq x \leq 13
 \end{aligned}$$

$$(d) \quad c = -1 \text{ or } c = -\frac{1}{4} \text{ (other answers possible)}$$

$$(e) \quad 3 - k < 0 \therefore k > 3$$

PAPER C

$$\begin{aligned}
 (a) \quad (1) \quad & (x-1)^2 = 2(1-x) \\
 & (x-1)^2 = -2(x-1) \\
 & (x-1)^2 + 2(x-1) = 0 \\
 & (x-1)(x-1+2) = 0 \\
 & (x-1)(x+1) = 0 \\
 & x = 1 \quad x = -1
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & 5^{-x} \cdot 5^{x-2} = \frac{25^{2x}}{5} \\
 & 5^{-x} \cdot 5^{x-2} = \frac{5^{4x}}{5^1} \\
 & 5^{-x+x-2} = 5^{4x-1} \\
 & -2 = 4x - 1 \\
 & x = -\frac{1}{4}
 \end{aligned}$$

PAPER D

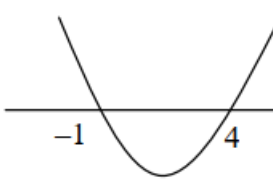
1.1	1.1.1	$ \begin{aligned} (2x-3)^2 &= 1 \\ 2x-3 &= \pm 1 \\ x &= \frac{3 \pm 1}{2} \\ x &= 1 \text{ or } x = 2 \\ \mathbf{OR} \\ 4x^2 - 12x + 9 &= 1 \\ 4x^2 - 12x + 8 &= 0 \\ 4(x-1)(x-2) &= 0 \\ x &= 1 \text{ or } x = 2 \end{aligned} $
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1.1.2	$2x^2 + 4x - 7 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-7)}}{2(2)}$ $= \frac{-4 \pm \sqrt{72}}{4}$ $= \frac{-2 \pm 3\sqrt{2}}{2}$
1.1.3	$x - \sqrt{2x-1} = 2$ $(x-2)^2 = (\sqrt{2x-1})^2$ $x^2 - 4x + 4 = 2x - 1$ $x^2 - 6x + 5 = 0$ $(x-1)(x-5) = 0$ $x \neq 1 \text{ or } x = 5$ $\therefore x = 5$

1.2.1	$x^2 + 5xy + 6y^2 = 0$ $(x+3y)(x+2y) = 0$ $x+3y=0 \quad \quad \quad x+2y=0$ $x=-3y \quad \text{OR} \quad \quad \quad x=-2y$ $\frac{x}{y} = -3 \quad \quad \quad \frac{x}{y} = -2$
	OR $x^2 + 5xy + 6y^2 = 0$ $x = \frac{-5y \pm \sqrt{(5y)^2 - 4(1)(6y^2)}}{2(1)}$ $x = \frac{-5y \pm \sqrt{y^2}}{2}$ $x = \frac{-5y \pm y}{2}$ $x = -3y \quad \quad \quad x = -2y$ $\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$

1.2.2	$\begin{array}{lcl} x + y = 8 & & x + y = 8 \\ -3y + y = 8 & & -2y + y = 8 \\ -2y = 8 & \text{OR} & -y = 8 \\ y = -4 & & y = -8 \\ x = 12 & & x = 16 \end{array}$
1.3.1	$\begin{array}{l} \sqrt{2p+5} = 0 \\ 2p+5 = 0 \\ 2p = -5 \\ p = -\frac{5}{2} \end{array}$
1.3.2	$\begin{array}{l} 2p+5 < 0 \\ p < -\frac{5}{2} \end{array}$

PAPER E

1.1.4	$\begin{array}{l} (x+1)(4-x) > 0 \\ (x+1)(x-4) < 0 \end{array}$ $\begin{array}{ccccccc} + & 0 & - & 0 & + & & \\ & -1 & & 4 & & & \end{array}$ $-1 < x < 4$	
1.2.1	$\begin{array}{l} 2^x + 2^{x+2} = -5y + 20 \\ 2^x(1 + 2^2) = -5y + 20 \\ 2^x = \frac{-5y+20}{5} \end{array}$	
1.2.2	$\begin{array}{l} \text{If } y = -4, \\ 2^x + 2^{x+2} = -5y + 20 \\ 2^x + 2^{x+2} = 40 \\ 2^x(1 + 2^2) = 40 \\ 2^x = 8 \\ 2^x = 2^3 \\ x = 3 \end{array}$	

1.2.3	$-y + 4 > 0$ $y < 4$ Largest integer value of y is 3 $2^x = -3 + 4$ $2^x = 1$ $x = 0$
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PAPER F

QUESTION 1

1.1.1 $(x-3)(x+4) = 18$
 $\therefore x^2 + x - 12 = 18 \checkmark$
 $\therefore x^2 + x - 30 = 0 \checkmark$
 $\therefore (x+6)(x-5) = 0 \checkmark$
 $\therefore x = -6 \text{ or } x = 5 \checkmark$

1.1.2 $x^2 = 6(x+2)$
 $\therefore x^2 = 6x + 12 \checkmark$
 $\therefore x^2 - 6x - 12 = 0 \checkmark$
 $\therefore x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-12)}}{2(1)} \checkmark$
 $\therefore x = \frac{6 \pm \sqrt{84}}{2}$
 $\therefore x = 7,58 \checkmark \text{ or } x = -1,58 \checkmark$

1.1.3 $2 - \frac{1}{x} = \frac{3}{x+2}$
 $\therefore 2x(x+2) - (x+2) = 3x \checkmark$
 $\therefore 2x^2 + 4x - x - 2 = 3x \checkmark \checkmark$
 $\therefore 2x^2 - 2 = 0 \checkmark$
 $\therefore x^2 - 1 = 0$
 $\therefore (x+1)(x-1) = 0 \checkmark$
 $\therefore x = -1 \text{ or } x = 1 \checkmark$

$$1.2 \quad y^2 + 2y - \frac{8}{y^2 + 2y} = 7$$

$$\text{Let } y^2 + 2y = k$$

$$\therefore k - \frac{8}{k} = 7 \quad \checkmark$$

$$\therefore k^2 - 7k - 8 = 0 \quad \checkmark$$

$$\therefore (k+1)(k-8) = 0 \quad \checkmark$$

$$\therefore k = -1 \text{ or } k = 8 \quad \checkmark$$

$$\therefore y^2 + 2y = -1 \text{ or } y^2 + 2y = 8$$

$$\therefore y^2 + 2y + 1 = 0 \text{ or } y^2 + 2y - 8 = 0 \quad \checkmark$$

$$\therefore (y+1)(y+1) = 0 \quad \checkmark \text{ or } (y+4)(y-2) = 0 \quad \checkmark$$

$$\therefore y = -1 \text{ or } y = -4 \text{ or } y = 2 \quad \checkmark$$

$$1.3 \quad 3x^2 - 6px - 9p^2 = 0$$

$$\therefore x^2 - 2px - 3p^2 = 0 \quad \checkmark$$

$$\therefore x^2 - 2px = 3p^2$$

$$\therefore x^2 - 2px + p^2 = 3p^2 + p^2 \quad \checkmark$$

$$\therefore (x-p)^2 = 4p^2 \quad \checkmark$$

$$\therefore x - p = \pm \sqrt{4p^2} \quad \checkmark$$

$$\therefore x = p \pm \sqrt{4p^2}$$

$$\therefore x = p + 2p \text{ or } x = p - 2p \quad \checkmark$$

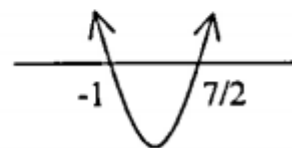
$$\therefore x = 3p \text{ or } x = -p \quad \checkmark$$

$$1.4 \quad 2x^2 - 5x \geq 7$$

$$\therefore 2x^2 - 5x - 7 \geq 0 \quad \checkmark$$

$$\therefore (x+1)(2x-7) \geq 0 \quad \checkmark$$

$$\therefore x \leq -1 \quad \checkmark \text{ or } x \geq \frac{7}{2} \quad \checkmark$$



QUESTION 2

2.1 $(2y+3)(x^2+4)=0$

$$\therefore y = -\frac{3}{2} \checkmark$$

or $x^2 = -4 \checkmark$

$\therefore$ no real value for $x \checkmark$

2.2 $2a - b = 7$

$$\therefore b = 2a - 7 \checkmark$$

Subs $b = 2a - 7$ into $a^2 + ab + b^2 = 7$

$$a^2 + a(2a - 7) + (2a - 7)^2 = 7 \checkmark$$

$$\therefore a^2 + 2a^2 - 7a + 4a^2 - 28a + 49 = 7 \checkmark \checkmark$$

$$\therefore 7a^2 - 35a + 42 = 0 \checkmark$$

$$\therefore a^2 - 5a + 6 = 0$$

$$\therefore (a - 2)(a - 3) = 0 \checkmark$$

$$\therefore a = 2 \text{ or } a = 3 \checkmark$$

$$\therefore b = -3 \text{ or } b = -1 \checkmark$$

2.3 $xy = 20 \checkmark$

$$\therefore y = \frac{20}{x}$$

$$(x+3)(y+1) = 40 \checkmark$$

$$\therefore xy + x + 3y + 3 = 40$$

Subs $y = \frac{20}{x}$ into $xy + x + 3y + 3 = 40$

$$\therefore x\left(\frac{20}{x}\right) + x + 3\left(\frac{20}{x}\right) + 3 = 40$$

$$\therefore 20 + x + \frac{60}{x} + 3 = 40$$

$$\therefore 20x + x^2 + 60 + 3x - 40x = 0$$

$$\therefore x^2 - 17x + 60 = 0 \checkmark$$

$$\therefore (x - 5)(x - 12) = 0$$

$$\therefore x = 5 \text{ or } x = 12 \checkmark$$

$$\therefore y = 4 \text{ or } y = \frac{5}{3} \checkmark$$

Dimensions $\therefore 5 \times 4 \text{ or } 12 \times \frac{5}{3}$

PATTERNS, SEQUENCES AND SERIES

PAPER A

4.1 The second, third, fourth and fifth terms are 1 ; - 6 ; T_4 and - 14

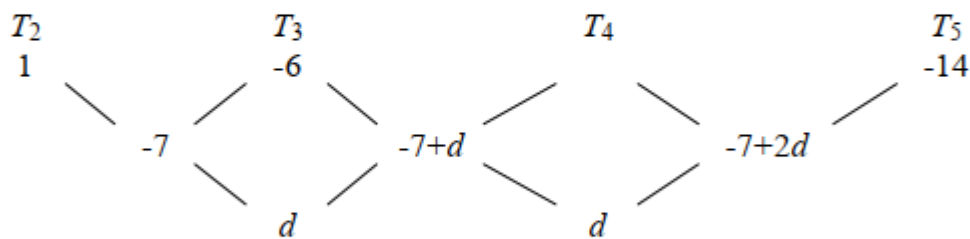
First differences are: - 7 ; $T_4 + 6$; $- 14 - T_4$

So $T_4 + 6 + 7 = - 14 - T_4 - 6$

$T_4 = - 11$

$d = - 11 + 6 + 7 = 2$ or $- 14 + 22 - 6 = 2$

OR



$$T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$$

$$-15 = (-7 + 2d) + (-7 + d) + -7$$

$$-15 = -21 + 3d$$

$$6 = 3d$$

$$d = 2$$

OR

$$4a + 2b + c = 1$$

$$9a + 3b + c = -6$$

$$5a + b = -7$$

$$25a + 5b + c = -14$$

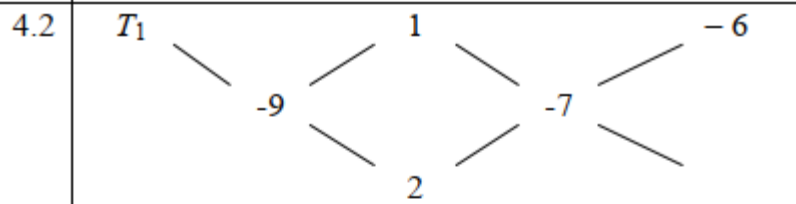
$$16a + 2b = -8$$

$$10a + 2b = -14$$

$$6a = 6$$

$$a = 1$$

$$d = 2a = 2$$



$$T_1 = 10$$

OR

$$a = 1$$

$$5a + b = -7$$

$$5(1) + b = -7$$

$$b = -12$$

$$a + b + c = 1$$

$$4(1) + 2(-12) + c = 1$$

$$c = 21$$

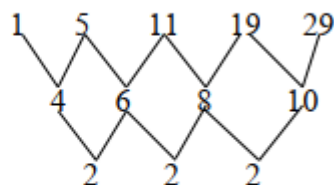
$$T_n = n^2 - 12n + 21$$

$$\begin{aligned} T_1 &= (1)^2 - 12(1) + 21 \\ &= 10 \end{aligned}$$

QUESTION/VRAAG 7

7.1	29
7.2	$T_n = an^2 + bn + c$ $1 = a + b + c$ $\therefore c = 1 - a - b$ $5 = 4a + 2b + c$ $5 = 4a + 2b + 1 - a - b$ $4 = 3a + b \quad (1)$ $11 = 9a + 3b + c$ $11 = 9a + 3b + 1 - a - b$ $\therefore 10 = 8a + 2b \quad (2)$ Solving (1) and (2) simultaneously. Los (1) en (2) gelyktydig op. $8 = 6a + 2b \quad (1) \times 2$ $10 = 8a + 2b \quad (2)$ $\therefore 2 = 2a$ $\therefore a = 1$ $\therefore b = 1$ $\therefore c = -1$ $T_n = n^2 + n - 1$

Or/Of



$$\begin{array}{lll}
 2a = 2 & 3a + b = 4 & a + b + c = 1 \\
 a = 1 & 3 + b = 4 & 1 + 1 + c = 1 \\
 & b = 1 & c = -1
 \end{array}$$

$$T_n = n^2 + n - 1$$

7.3	$T_n = n^2 + n - 1$ or/of $T_n = 100(101) - 1$ $\therefore T_{100} = 100^2 + 100 - 1 = 10\,099$ $= 10\,099$
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PAPER B

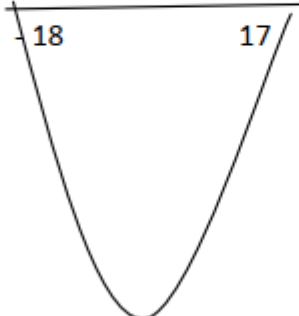
QUESTION 4

4.1	$\begin{array}{cccc} -7 & 0 & 9 & 20 \\ & 7 & 9 & 11 \\ & & 2 & 2 \end{array}$ $2a = 2$ $a = 1$ $3(1) + b = 7$ $b = 4$ $(1) + (4) + c = -7$ $c = -12$ $\therefore T_n = n^2 + 4n - 12$ OR $2a = 2$ $a = 1$ $T_2 = 2^2 + b(2) + c = 0$ $2b + c = -4 \quad (1) \quad 3(1) + b = 7$ $T_3 = 3^2 + b(3) + c = 9$ $3b + c = 0 \quad (2) \quad \text{OR} \quad b = 4$ $1 + a + c = -7$ $c = -12$
	$(2) - (1) \quad b = 4$ $\therefore c = -4 - 2(4) = -12$ $T_n = n^2 + 4n - 12$
4.2	$n^2 + 4n - 12 = 128$ $n^2 + 4n - 140 = 0$ $(n + 14)(n - 10) = 0$ $n \neq -14 \text{ or } n = 10$ $\text{invalid} \quad \therefore n = 10$

4.3	$-7 ; 0 ; 9 ; 20 ; \dots$ first difference $7 \quad 9 \quad 11$ second difference $2 \quad 2$ $F_n = 2n + c$ $F_1 = 2(1) + c = 7$ $\therefore c = 5$ $F_n = 2n + 5$
4.4	$F_n = 2n + 5 = 599$ $2n = 594$ $\therefore n = 297$ this difference will be between term 297 and term 298

QUESTION 5

5.1	<table><tr><td>Pattern</td><td>1</td><td>2</td><td>3</td></tr><tr><td>White squares</td><td>4</td><td>12</td><td>24</td></tr></table>	Pattern	1	2	3	White squares	4	12	24
	Pattern	1	2	3					
	White squares	4	12	24					
40									
5.2	$W_n = 2n^2 + 2n$ $W_{157} = 2(157)^2 + 2(157)$ $= 49612$								

5.3	$2n^2 + 2n + 1 < 613$ $2n^2 + 2n - 612 < 0$ $n^2 + n - 306 < 0$ $(n - 17)(n + 18) < 0$  $\therefore n = 16$
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5.4	$P_n = 4n^2 + 4n + 1$ $= (2n)^2 + 2(2n) + 1$ $2n \text{ is even for all } n \in \mathbb{Z}$ $\therefore \text{Total squares used in the } n^{\text{th}} \text{ pattern is always odd.}$ OR $P_n = 4n^2 + 4n + 1$ $= 2(2n^2 + 2n) + 1$ $2(2n^2 + 2n) \text{ is odd for all } n \in \mathbb{Z}$ $2(2n^2 + 2n) + 1 \text{ is odd for all } n \in \mathbb{Z}$ $\therefore \text{Total squares used in the } n^{\text{th}} \text{ pattern is always odd.}$
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PAPER C

QUESTION 3

3.1	$T_2 - T_1 = T_3 - T_2$ $p + 5 - 2p + 3 = 2p + 7 - p - 5$ $-p + 8 = p + 2$ $p = 3$
3.2	Pattern is 3 ; 8 ; 13 ; ... $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{120} = \frac{120}{2}[2(3) + 119(5)]$ $= 36060$
3.3.1	$x = k + 1 \text{ and } y = k + 2$
3.3.2	$T_x = a + (x-1)d = 3 + 5k$ $T_y = a + (k+1)d$ $= 3 + (k+1)(5)$ $= 8 + 5k$ $T_x + T_y = 11 + 10k$

QUESTION 4

4.1.1	15 ; 5
4.1.2	$S_{\infty} = \frac{a}{1-r}$ $= \frac{15}{1-\frac{1}{3}}$ $= \frac{45}{2} = 22,5$

4.2	$\sin 30^{\circ}; \cos 30^{\circ}; \frac{3}{2}$ $\frac{1}{2}; \frac{\sqrt{3}}{2}; \frac{3}{2}$ $a = \frac{1}{2}; r = \sqrt{3}$ $ar^{n-1} = 40,5\sqrt{3}$ $\frac{1}{2}(\sqrt{3})^{n-1} = \frac{81}{2}\sqrt{3}$ $3^{\frac{n-1}{2}} = 3^4 \cdot 3^{\frac{1}{2}}$ $\frac{n-1}{2} = 4\frac{1}{2} = \frac{9}{2}$ $n-1 = 9$ $n = 10$
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OR

	$\sin 30^{\circ}; \cos 30^{\circ}; \frac{3}{2}$ $\frac{1}{2}; \frac{\sqrt{3}}{2}; \frac{3}{2}$ $a = \frac{1}{2}; r = \sqrt{3}$ $ar^{n-1} = \frac{81}{2}\sqrt{3}$ $\frac{1}{2}(\sqrt{3})^{n-1} = \frac{81}{2}\sqrt{3}$ $\frac{(\sqrt{3})^n}{\sqrt{3}} = 81\sqrt{3}$ $243 = (\sqrt{3})^n$ $3^5 = 3^{\frac{1}{2}n}$ $n = 10$
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PAPER D

QUESTION 3

3.1

$$r = \frac{2(3x-1)^2}{2(3x-1)}$$

$$r = 3x - 1$$

$$-1 < 3x - 1 < 1$$

$$0 < 3x < 2$$

$$0 < x < \frac{2}{3}$$

3.2

$$T_2 = ar$$

$$6 = kr$$

$$r = \frac{6}{k} \quad \dots (1)$$

sub. (1) into (2)

$$25 = \frac{k}{1 - \frac{6}{k}}$$

$$k = 25 \left(1 - \frac{6}{k} \right)$$

$$k = 25 - \frac{150}{k}$$

$$0 = k^2 - 25k + 150$$

$$0 = (k - 10)(k - 15)$$

$$\therefore k = 10 \text{ or } k = 15$$

$$S_{\infty} = 25$$

$$S_{\infty} = \frac{a}{1-r}$$

$$25 = \frac{k}{1-r} \quad \dots (2)$$

3.3

$$(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \dots + (81 \times 82)$$

$$(4n - 3) \times (4n - 2)$$

$$= 16n^2 - 20n + 6$$

$$4n - 3 = 81$$

OR

$$4n - 2 = 82$$

$$4n = 84$$

$$4n = 84$$

$$n = 21$$

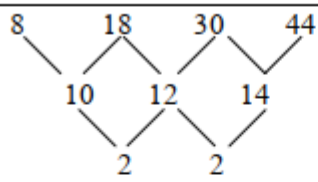
$$n = 21$$

$$\sum_{n=1}^{21} 16n^2 - 20n + 6 \quad \text{OR} \quad \sum_{n=1}^{21} (4n - 3)(4n - 2)$$

PAPER E

QUESTION 2

2.1.1	$\frac{1}{16}; 13$
2.1.2	$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \text{to } 25 \text{ terms}\right) \quad (4 + 7 + 10 + 13 + \dots \text{to } 25 \text{ terms})$ $\frac{a(r^n - 1)}{r - 1} = \frac{n}{2}[2a + (n - 1)d]$ $= \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25} - 1\right)}{\frac{1}{2} - 1} = \frac{25}{2}[2(4) + 24(3)]$ $= 0,9999999 \quad = 1\,000$ $S_{50} = 1001,00$
	OR $S_{50} = 25 \text{ terms of } 1^{\text{st}} \text{ sequence} + 25 \text{ terms of } 2^{\text{nd}} \text{ sequence}$ $S_{50} = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \text{to } 25 \text{ terms}\right) + (4 + 7 + 10 + 13 + \dots \text{to } 25 \text{ terms})$ $S_{50} = \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25} - 1\right)}{\frac{1}{2} - 1} + \frac{25}{2}[2(4) + 24(3)]$ $S_{50} = 0,9999999\dots + 1000$ $S_{50} = 1001,00$

2.2.1	60 ; 78
2.2.2	 $2a = 2$ $a = 1$ $T_n = n^2 + bn + c$ $8 = 1 + b + c$ $7 = b + c \quad \dots(i)$ $18 = 4 + 2b + c$ $14 = 2b + c \quad \dots(ii)$ $(ii) - (i): \quad 14 = 2b + c$ $7 = b + c$ $\therefore 7 = b$ $c = 0$ $T_n = n^2 + 7n$
	OR
	$T_1 = 8$ $T_2 - T_1 = 10$ $T_3 - T_2 = 12$ $T_n - T_{n-1} = n\text{th term of sequence with } a = 8 \text{ and } d = 2$ Add both sides $T_n = 8 + 10 + 12 + \dots + \text{to } 25 \text{ terms}$ $T_n = \frac{n}{2}[16 + 2(n-1)]$ $T_n = n(n+7)$
2.2.3	$n(n+7) = 330$ $n^2 + 7n - 330 = 0$ $(n+22)(n-15) = 0$ $n = -22 \text{ or } n = 15$ $n = 15$ $\therefore 15^{\text{th}} \text{ term is } 330.$

2.3	$S_n = a + [a + d] + [a + 2d] + \dots + [a + (n-2)d] + [a + (n-1)d]$ $S_n = [a + (n-1)d] + [a + (n-2)d] + \dots + [a + d] + a$ $2S_n = [2a + (n-1)d] + [2a + (n-1)d] + \dots + [2a + (n-1)d] + [2a + (n-1)d]$ $= n[2a + (n-1)d]$ $S_n = \frac{n}{2}[2a + (n-1)d]$
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QUESTION 3

3.1	$T_n = (8x^2) \left(\frac{x}{2} \right)^{n-1}$
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3.2	$\text{ratio} = \frac{x}{2}$ $-1 < \frac{x}{2} < 1$ $-2 < x < 2$
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3.3	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8x^2}{1 - \frac{x}{2}}$ $S_\infty = \frac{8\left(\frac{3}{2}\right)^2}{1 - \frac{1}{2}\left(\frac{3}{2}\right)}$ $S_\infty = 72$
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OR

	$18 + \frac{27}{2} + \frac{81}{8} + \dots$ $S_\infty = \frac{18}{1 - \frac{3}{4}}$ $S_\infty = \frac{18}{\frac{1}{4}}$ $S_\infty = 72$
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PAPER F

QUESTION 2

2.1	$20 ; 24 ; 28 ; 32 ; \dots$ $4 \quad 4 \quad 4$ $T_n = 20 + (n - 1) 4$ $100 = 20 + 4n - 4$ $4n = 84$ $n = 21$ On the 21st day she will cycle 100 km.
	OR $T_n = 4n + 16$ $100 = 4n + 16$ $4n = 84$ $n = 21$ On the 21st day she will cycle 100 km. OR $100 = 20 + 80$ $= 20 + 4(21 - 1)$ $\therefore n = 21$
2.2	$S_n = \frac{n}{2}[2a + (n - 1)d]$ $s_{14} = \frac{14}{2}[2(20) + (14 - 1)4]$ $= 644 \text{ km}$
2.3	No. It will not be humanly possible to just keep on increasing the distance covered indefinitely. For example: $T_{1000} = 4\,016$ km in one day.

QUESTION 3

3.1	45
3.2	$T_n = an^2 + bn + c$ Second difference of terms is 2. $a = 1$ $3a + b = 7$ $3 + b = 7$ $b = 4$ $a + b + c = 5$ $1 + 4 + c = 5$ $c = 0$ $T_n = n^2 + 4n$
	OR $T_n = an^2 + bn + c$ Second difference of terms is 2. $a = 1$ $T_0 = 0 = c$ $T_n = n^2 + bn + 0$ $5 = (1)^2 + (1)b$ $b = 4$ $T_n = n^2 + 4n$
	OR If $T_n = an^2 + bn + c$ $5 = T_1 = a + b + c$ $12 = T_2 = 4a + 2b + c$ $21 = T_3 = 9a + 3b + c$ $\Rightarrow 3a + b = 7$ $\Rightarrow b = 4$ $c = 0$

QUESTION 4

4.1	$S = a + ar + ar^2 + \dots + ar^{n-1}$ $rS = ar + ar^2 + \dots + ar^{n-1} + ar^n$ $S - rS = a - ar^n$ $S(1-r) = a(1-r^n)$ $S = \frac{a(1-r^n)}{1-r}$
4.2.1	$15; 5; \frac{5}{3}; \dots$ $r = \frac{5}{15} = \frac{1}{3}$ The series converges because $-1 < r < 1$
4.2.2	$S_{\infty} = \frac{15}{1 - \frac{1}{3}}$ $= \frac{45}{2}$
4.3.1	$S_{24} = 2^{24+2} - 4$ $= 67108860$
4.3.2	$S_{24} = 2^{24+2} - 4 = 67108860$ $S_{23} = 2^{23+2} - 4 = 33554428$ $T_{24} = 33554432$ OR $T_{24} = S_{24} - S_{23}$ $= 2^{26} - 2^{25}$ $= 2 \times 2^{25} - 2^{25}$ $= 2^{25}$

FUNCTIONS AND GRAPHS

PAPER A

QUESTION 5

5.1

$$\begin{aligned}x &= -\frac{b}{2a} \\&= -\frac{1}{2(-2)} \\&= \frac{1}{4} \\ \therefore y &= -2\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right) + 6 \\ y &= \frac{49}{8}\end{aligned}$$

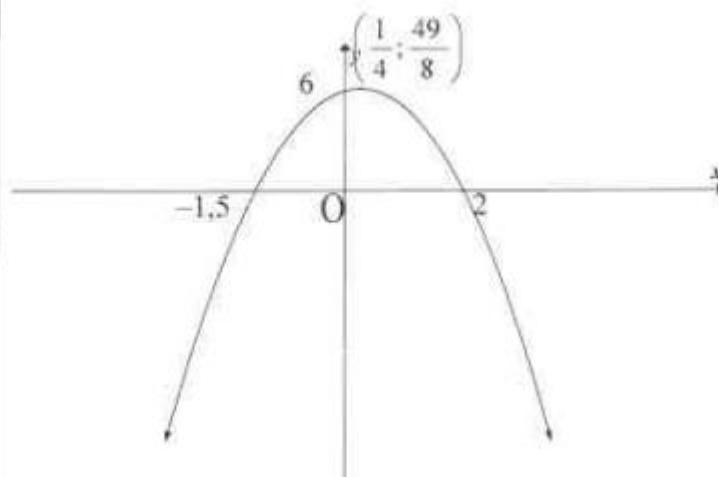
5.2

$$\begin{aligned}y &= -2(0)^2 + 0 + 6 \\ \therefore y \text{ intercept } &(0;6)\end{aligned}$$

5.3

$$\begin{aligned}&\text{x intercepts} \\ 0 &= -2x^2 + x + 6 \\ 0 &= 2x^2 - x - 6 \\ 0 &= (2x + 3)(x - 2) \\ \therefore x &= 2 \text{ or } x = -\frac{3}{2} \\ (2;0) &\text{ and } \left(-\frac{3}{2};0\right)\end{aligned}$$

5.4



5.5	$k = \frac{49}{8}$
5.6	New/Nuwe turning point/ <i>drpn.t</i> $\left(\frac{9}{4}; \frac{57}{8}\right)$ Equation/ <i>verg.</i> of h $y = -2\left(x - \frac{9}{4}\right)^2 + \frac{57}{8}$

QUESTION/VRAAG 6

6.1	$x = -3$ and $y = -1$
6.2	$x \in R ; x \neq -3$ OR $x \in (-\infty; -3) \cup (-3; \infty)$

6.3.1	At B, $x = 0$ $\therefore y = \frac{1}{0+3} - 1$ $y = -\frac{2}{3}$ $\therefore OB = \frac{2}{3}$ units
6.3.2	At A, $y = 0$ $0 = \frac{1}{x+3} - 1$ $1 = \frac{1}{x+3}$ $x+3 = 1$ $x = -2$ $\therefore OA = 2$ units/ eenhede

6.4	$\frac{1}{x+3} - 1 = \frac{1}{2}x$ $2 - 2(x+3) = x(x+3)$ $x^2 + 3x - 2 + 2x + 6 = 0$ $x^2 + 5x + 4 = 0$ $(x+4)(x+1) = 0$ $x = -4 \text{ or / of } x = -1$ when / wanneer $x = -1$; $y = -\frac{1}{2}$ when / wanneer $x = -4$; $y = -2$ $\therefore C \left(-1; -\frac{1}{2}\right) \text{ and } D (-4; -2)$
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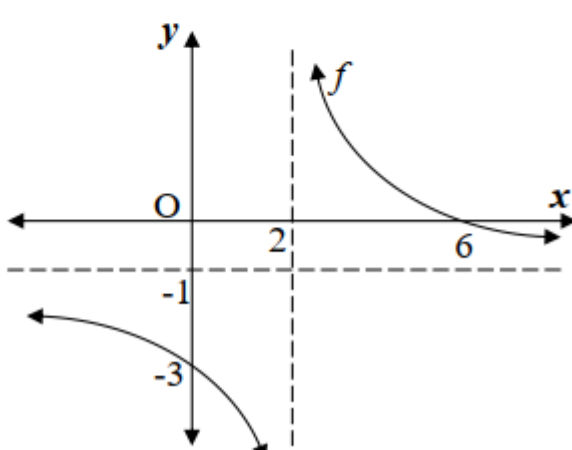
6.5	$\frac{1}{x+3} \geq \frac{x+2}{2}$ $\frac{1}{x+3} \geq \frac{x}{2} + 1$ $\frac{1}{x+3} - 1 \geq \frac{x}{2}$ $\therefore f(x) \geq g(x)$ $\therefore x \leq -4 \text{ or } -3 < x \leq -1$
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QUESTION/VRAAG 7

7.1	$q = 2$ $f(x) = 2 \cdot b^{x+1} + 2$ $20 = 2 \cdot b^{1+1} + 2$ $18 = 2 \cdot b^2$ $9 = b^2$ $b = 3$ $f(x) = 2 \cdot 3^{x+1} + 2$
7.2	$y = 2 \cdot 3^{-1+1} + 2$ $y = 2 \cdot 1 + 2$ $y = 4$
7.3	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{20 - 4}{1 - (-1)}$ $= 8$
7.4	$h(x) = -2 \cdot 3^{x+1} + 2$
7.5	$y < 2$

PAPER B

QUESTION 5

5.1	$x = 2$ and $y = -1$
5.2	y - intercept: $(0; -3)$ x - intercept: $\frac{-4}{2-x} - 1 = 0$ $\frac{-4}{2-x} = 1$ $-4 = 2 - x$ $x = 6$ $(6; 0)$
5.3	

QUESTION 6

6.1	$A(0; 6)$
6.2	$x = -\frac{b}{2a} = 2,5$ $S(5; 6)$
6.3	$-x^2 + 5x + 6 = 0$ $x^2 - 5x - 6 = 0$ $(x + 1)(x - 6) = 0$ $x = -1$ or $x = 6$ $B(-1; 0), C(6; 0)$
6.4	$(-x^2 + 5x + 6) - (x + 1) = 5$ $-x^2 + 5x + 6 - x - 1 = 5$ $x^2 - 4x = 0$ $x(x - 4) = 0$ $x = 0$ or 4 $OR = 4$ units

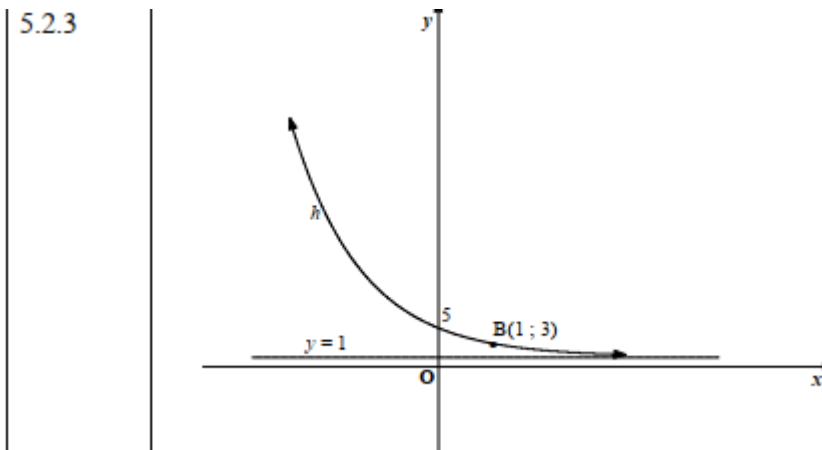
6.5.1	$x = \frac{-1+6}{2} = \frac{5}{2}$ $y = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) + 6 = \frac{49}{4} = 12,25$ $\left(\frac{5}{2}; 12,25\right)$
6.5.2	$PQ = -x^2 + 4x + 5$ $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$ $\text{Max. PQ} = -(2)^2 + 4(2) + 5 = 9\text{units}$

QUESTION 7

7.1	$y = 5^x$
7.2	$y > 0$ or $y \in (0; \infty)$
7.3	$\log_5 x = -4$ $x = 5^{-4} = \frac{1}{625}$ $0 < x \leq \frac{1}{625}$

PAPER C

5.2.1	For y-intercept/ <i>Vir y-afsnit</i> substitution $x = 0$: $y = 4 \cdot 2^0 + 1$ $= 5$ $H(0; 5)$
5.2.2	For x-intercept/ <i>Vir y-afsnit</i> $y = 0$ i.e./d.i. $4 \cdot 2^{-x} + 1 = 0$ $4 \cdot 2^{-x} = -1$ $2^{-x} = -\frac{1}{4}, \text{ which is impossible, since } 2^{-x} > 0 \text{ for } x \in R$, wat onmoontlik is omdat $2^{-x} > 0$ vir $x \in R$ Therefore/ <i>Dus</i> : no solution/ <i>geen oplossing</i> , which means there will be no x-intercept/ <i>wat beteken daar sal geen x-afsnit wees nie</i> .



5.2.4	$g(x) = 4(2^{-x} + 2)$ $= 4 \cdot 2^{-x} + 8$ The graph of h is translated 7 units upwards to form g/ <i>Die grafiek van h word 7 eenhede na bo getransleer om g te vorm.</i>
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QUESTION 6

6.1	$A(0; 2)$
6.2	$-x^2 + x + 2 = \frac{1}{2}x^2 - x$ $0 = \frac{3}{2}x^2 - 2x - 2$ $0 = 3x^2 - 4x - 4$ $(3x + 2)(x - 2) = 0$ $x = -\frac{2}{3} \quad \text{or} \quad x = 2$ $y = \frac{8}{9} \quad \text{or} \quad y = 0$ $C\left(-\frac{2}{3}; \frac{8}{9}\right) \quad \& \quad D(2; 0)$
6.3	$x \leq -\frac{2}{3} \text{ or } x \geq 2$

6.4	Length of PQ = $-x^2 + x + 2 - \left(\frac{1}{2}x^2 - x\right)$ $L = -\frac{3}{2}x^2 + 2x + 2$ $x = -\frac{b}{2a} = -\frac{2}{2\left(-\frac{3}{2}\right)} = \frac{2}{3}$ OR $L' = -3x + 2 = 0 \therefore x = \frac{2}{3}$ Maximum value of PQ $= -\frac{3}{2}\left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) + 2$ $= \frac{8}{3} = 2\frac{2}{3} = 2,67 \text{ units}$
6.5	$f(x) = -x^2 + x + 2$ $f'(x) = -2x + 1 = 3$ $x = -1$

6.6	Axis of symmetry: $x = \frac{1}{2}$ Maximum value: $y = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 2 = 2\frac{1}{4}$ $2 < k < 2\frac{1}{4}$
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QUESTION/VRAAG 7

7.1	$CD = 2x + 3 - (-2x^2 + 14x + k)$ $= 2x + 3 + 2x^2 - 14x - k$ $= 2x^2 - 12x + 3 - k$
7.2	Minimum value occurs at/Minimum waarde vind plaas by $x = \frac{-b}{2a}$ $= \frac{12}{2(2)}$ $= 3$ Minimum value/Minimum waarde $5 = 2(3)^2 - 12(3) + 3 - k$ $5 = 18 - 36 + 3 - k$ $k = -20$

PAPER D

QUESTION 6

6.1	$f(x) = -2x^2 - 5x + 3$ $x = \frac{-(-5)}{2(-2)}$ $x = -\frac{5}{4}$ $f\left(-\frac{5}{4}\right) = -2\left(-\frac{5}{4}\right)^2 - 5\left(-\frac{5}{4}\right) + 3$ $f\left(-\frac{5}{4}\right) = \frac{49}{8}$ $\therefore TP\left(-\frac{5}{4}; \frac{49}{8}\right)$
6.2	$y \in \left(-\infty; \frac{49}{8}\right]$ OR $y \leq \frac{49}{8}$
6.3	$\tan 135^\circ = -1$ $m = -1$ $f'(x) = -1$ $-4x - 5 = -1$ $\therefore x = -1$ $f(-1) = -2(-1)^2 - 5(-1) + 3$ $y = 6$ $\therefore P(-1; 6)$
6.4	$k < -\frac{49}{8} \text{ or } k > -\frac{49}{8}$

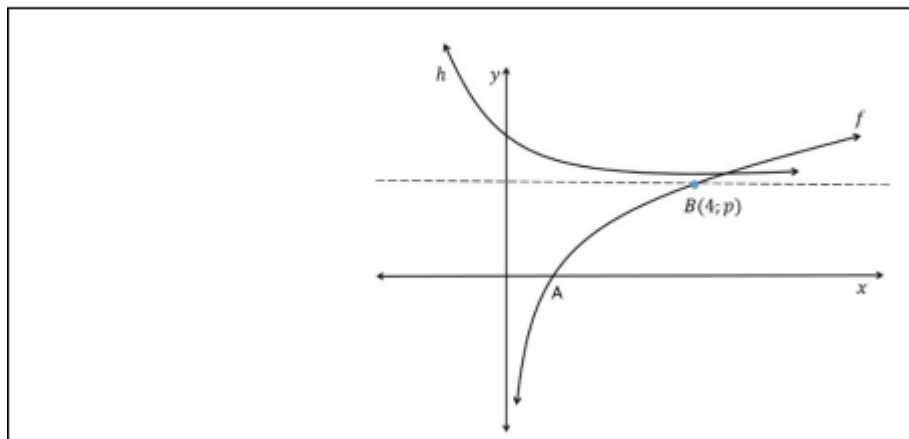
QUESTION 7	
7.1	$f(x) = a^x$ $\frac{1}{4} = a^2$ $\sqrt{\frac{1}{4}} = a$ $\frac{1}{2} = a$
7.2	$y = \left(\frac{1}{2}\right)^x$ $x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$
7.3	$y = \left(\frac{1}{2}\right)^x$ $-y = \left(\frac{1}{2}\right)^x$ $h(x) = -\left(\frac{1}{2}\right)^x$ OR $h(x) = -a^x$ OR $h(x) = -f(x)$
7.4	$x \leq 0$ OR $x > 0$ OR $x < 0$ OR $x \geq 0$

QUESTION 8		
8.1	8.1.1	$x < 2$
	8.1.2	$0 < x \leq 1$
8.2	8.2.1	$g(x) = \log_2 x$ $y = \log_2 x$ $\therefore x = 2^y$ $\therefore y = 2^x$ $\therefore g^{-1}(x) = 2^x$
	8.2.2	$\log_2(3-x) = x$ $2^x = 3-x$ therefore point of intersection of g^{-1} and f
	8.2.3	$x = 1$

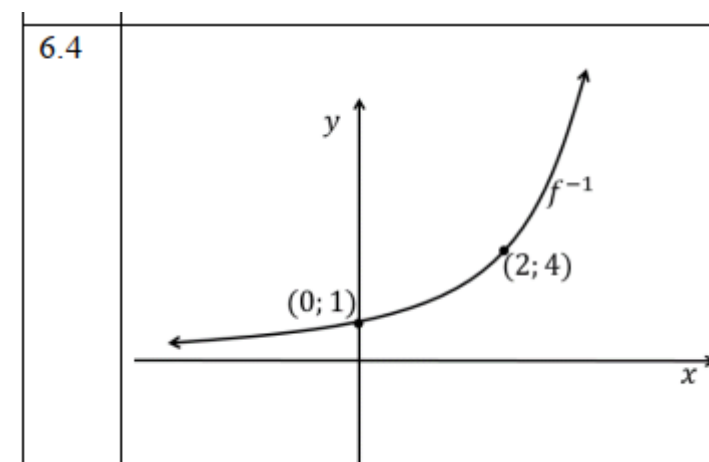
PAPER E

5.1	$g(x) = -3x + 20$ $g(3) = -3(3) + 20 = 11$ $\therefore k = 11$
5.2	$y \geq -11$ OR $x \in [-11; \infty)$
5.3	$y = a(x - 3)^2 + 11$ subst. (6; 2) $2 = a(6 - 3)^2 + 11$ $-9 = 9a$ $-1 = a$ $y = -1(x - 3)^2 + 11$ $y = -(x^2 - 6x + 9) + 11$ $y = -x^2 + 6x + 2$ $\therefore a = -1 ; b = 6 \text{ and } c = 2$
5.4	$3 < x < 6$ OR $x \in (3; 6)$
5.5	• Real <i>Reëel</i> • Rational <i>Rasionaal</i> • Equal <i>Gelyk</i>
5.6	$x > 3$ OR $x \in (3; \infty)$

QUESTION 6



6.1	$A(1; 0)$
6.2	$x \in \mathbb{R} ; x > 0$ or $x \in (0; \infty)$
6.3	$x = \log_2 y$ $y = 2^x$



6.5	$p = \log_2 4 = 2$ $\therefore y = 2$
6.6	Reflection in the y -axis and translate 2 units down.

PAPER F

QUESTION 5

5.1.1	$C(0; -3)$
5.1.2	$f(x) = x^2 - 2x - 3$ $(x-3)(x+1) = 0$ $x = -1$ or $x = 3$ $AB = 3 - (-1)$ $AB = 4$ units
5.1.3	$x = \frac{2}{2(1)}$ or $2x - 2 = 0$ or $x = \frac{-1+3}{2}$ $= 1$ $y = (1)^2 - 2(1) - 3$ $= -4$ $D(1; -4)$
5.1.4	$C(0; -3)$ $D(1; -4)$ Average gradient / <i>Gemiddelde gradiënt</i> $= \frac{-4+3}{1-0}$ or $\frac{-3+4}{0-1}$ $= -1$
5.1.5	$OC = OB = 3$ $\hat{OCB} = 45^\circ$ isosceles right angled triangle <i>Gelykbenige reghoekige driehoek</i> OR / OF $\tan \beta = m_g$ $\tan \beta = 1$ $\beta = 45^\circ$ $\hat{OBC} = 45^\circ$ $\hat{OCB} = 45^\circ$
5.1.6	$-4 < k < -3$ OR $(-4; -3)$

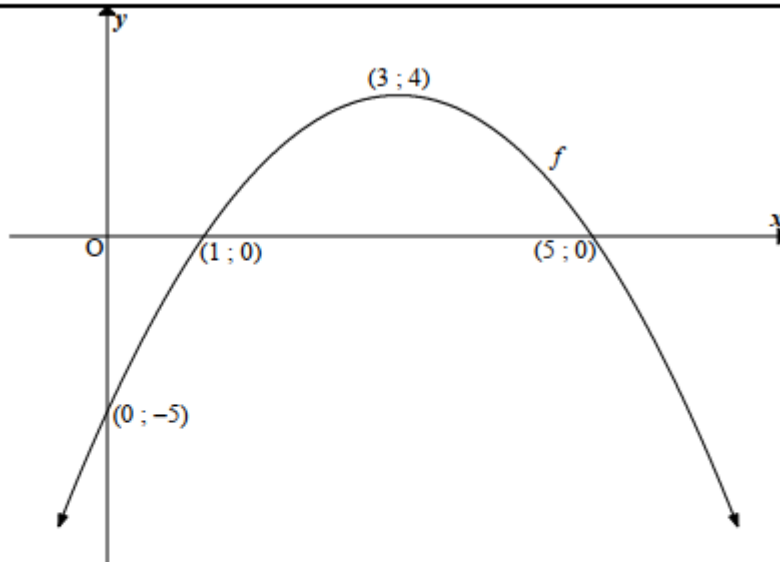
$$5.1.7 \quad f'(x) \cdot f''(x) > 0$$

$$(2x - 2) \cdot 2 > 0$$

$$2x - 2 > 0$$

$$x > 1$$

5.2



$$f(x) = a(x - 1)(x - 5)$$

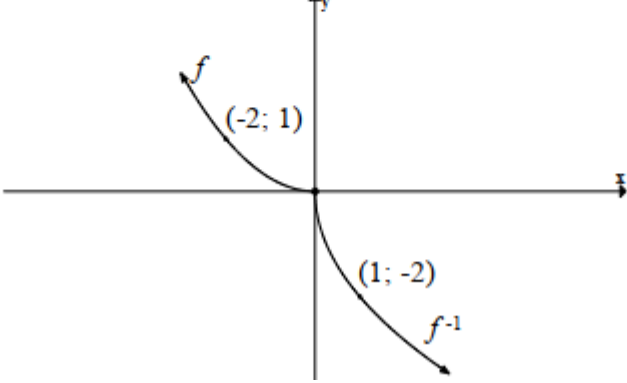
$$4 = a(3 - 1)(3 - 5)$$

$$4 = -4a$$

$$a = -1$$

$$f(x) = -x^2 + 6x - 5$$

QUESTION/VRAG 6

6.1	$f: y = \frac{1}{4}x^2$ $f^{-1}: x = \frac{1}{4}y^2$ $y^2 = 4x$ $y = \pm\sqrt{4x}$ $f^{-1}(x) = -\sqrt{4x} \quad \text{OR/OF} \quad f^{-1}(x) = -2\sqrt{x}$
6.2	
6.3	Yes. No value of x in the domain of f^{-1} maps onto more than one y-value. <i>Ja. Geen waarde van x in die definisieversameling van f^{-1} assosieer met meer as een y-waarde nie.</i> OR/OF Yes. One to one function./Ja. Een-tot-een-funksie. OR/OF Yes. Vertical line test holds./Ja. Die vertikale lyntoets werk.

FINANCE, GROWTH AND DECAY

PAPER A

7.1	$A = P(1 + i)^n$ $23000 = 1570(1.12)^n$ $(1.12)^n = 14,64968153..$ $n \log(1.12) = \log 14,64968153..$ $n = 23,69 \text{ years}$ or $n = 24 \text{ years}$ or $n = 23 \text{ years } 8 \text{ months}$ or $n = 23,7 \text{ years}$
7.2.1	$A = P(1 + i)^n$ $= 800000(1.08)^5$ $= R1175462,46$ $\therefore R1175462,46 - R200\,000$ $= R975462,46$ Some calculators give R 975 462,50
7.2.2	$F = \frac{x[(1 + i)^n - 1]}{i}$ $975462,46 = x \frac{[1,01]^{60} - 1}{0,01}$ $\frac{975462,46 \times 0,01}{[1,01]^{60} - 1} = x$ $x = R\,11944,00$

7.2.3	$Service = [5000(1,01)^{48} + 5000(1,01)^{36} + 5000(1,01)^{24} + 5000(1,01)^{12} + 5000]$ $= 32197,77$ $975462,46 = x \frac{[1,01]^{60} - 1}{0,01} - Service$ $975462,46 = 81,66966986x - 32197,77$ $x = R\ 12338,24$ OR $Service = \frac{5000[1,01^{60} - 1]}{1,01^{12} - 1}$ $= 32197,77$ $975462,46 = x \frac{[1,01]^{60} - 1}{0,01} - Service$ $975462,46 = 81,66966986x - 32197,77$ $x = R\ 12338,24$
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PAPER B

QUESTION 7

7.1	$A = P(1 - i)^n$ $\frac{P}{2} = P(1 - 0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log \frac{1}{2} = n \log 0,93$ $n = \frac{\log \frac{1}{2}}{\log 0,93}$ $= 9,55 \text{ years}$ OR $A = P(1 - i)^n$ $\frac{P}{2} = P(1 - 0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log_{0,93} \frac{1}{2} = n$ $n = 9,55 \text{ years}$
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7.2	Radesh: $A = P(1 + in)$ $= 6\,000(1 + 0,085 \times 5) \quad \text{OR}$ $= 8\,550$ $A = 6\,000 + 8,5\% \text{ of } 6000 \times 5$ $= 6000 + 510 \times 5$ $= 6000 + 2550$ $= 8\,550$ Bonus = $0,05 \times 6\,000$ $= 300$ Received = $8\,550 + 300$ $= \text{R}8\,850$ Thandi: $A = P(1 + i)^n$ $= 6\,000 \left(1 + \frac{0,08}{4}\right)^{20}$ $= \text{R}8\,915,68$ Thandi's investment is bigger.
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7.3	F_v = initial deposit with interest + annuity $= 1\,000 \left(1 + \frac{0,15}{12}\right)^{18} + 700 \left(\frac{\left(1 + \frac{0,15}{12}\right)^{18} - 1}{\frac{0,15}{12}} \right)$ $= 1\,250,58 + 14\,032,33$ $= \text{R}15\,282,91$
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PAPER C

QUESTION 8

8.1	Depreciation value = $7\,200(1 - 0,25)^3$ $= \text{R}3\,037,50$
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8.2.1	$300\,000 = \frac{5\,000[1 - (1,015)^{-n}]}{0,015}$ $4\,500 = 5\,000 - 5\,000(1,015)^{-n}$ $5\,000(1,015)^{-n} = 500$ $(1,015)^{-n} = 0,1 \quad \text{or} \quad (1,015)^n = 10$ $-n = \frac{\log 0,1}{\log 1,015}$ $n = 154,65$ Number of payments = 155
8.2.2	Balance outstanding $= 300\,000 \left(1 + \frac{0,18}{12}\right)^{154} - \frac{5\,000 \left[\left(1 + \frac{0,18}{12}\right)^{154} - 1 \right]}{\frac{0,18}{12}}$ $= \text{R}3\,230,50$
8.2.3	Amount paid in last month $= 3\,230,50 \left(1 + \frac{0,18}{12}\right)$ $= \text{R}3\,278,96$
8.2.4	Total repaid = $(154 \times 5\,000) + 3\,278,96 = \text{R}773\,278,96$

PAPER D

QUESTION 8

8.1	$A = P(1 - i)^n$ $250\,000 = P(1 - 13,5\%)^5$ $P = \frac{250\,000}{(1 - 13,5\%)^5}$ $= \text{R}516\,249$
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8.2.1	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $950000 = \frac{x \left[1 - \left(1 + \frac{14,25\%}{12} \right)^{-240} \right]}{\frac{14,25\%}{12}}$ $x = R11986,33$
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8.2.2	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $= \frac{11986,33 \left[1 - \left(1 + \frac{14,25\%}{12} \right)^{-140} \right]}{\frac{14,25\%}{12}}$ $= R816048,67$ OR $A = P(1 + i)^n$ $A = 950\,000 \left(1 + \frac{14,25\%}{12} \right)^{100}$ $= R3093215,766$ $F = \frac{x[(1 + i)^n - 1]}{i}$ $F = \frac{11986,33 \left[\left(1 + \frac{14,25\%}{12} \right)^{100} - 1 \right]}{\frac{14,25\%}{12}}$ $= R2277167,107$ Balance on Loan $= R3093215,766 - R2277167,107$ $= R816048,67$
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8.2.3	$A = P(1 + i)^n$ $= 816\,048,67 \left(1 + \frac{14,25\%}{12} \right)^4$ $= R855\,506,92$ $855\,506,92 = \frac{x[1 - (1 + i)^{-n}]}{i}$ $= \frac{x \left[1 - \left(1 + \frac{14,25\%}{12} \right)^{-136} \right]}{\frac{14,25\%}{12}}$ $x = R12711,51$
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PAPER E

QUESTION 8

8.1	$A = P(1 + i)^n$ $3P = P \left(1 + \frac{i}{12}\right)^{72}$ $i = 12(\sqrt[72]{3} - 1)$ $i = 0,1845$ Annual interest rate is 18,45 % p.a.
8.2.1	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $192\,000 = \frac{x \left[1 - \left(1 + \frac{0,12}{12}\right)^{-60}\right]}{\frac{0,12}{12}}$ $x = R4270,93$
8.2.2	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $= \frac{4270,93 \left[1 - \left(1 + \frac{0,12}{12}\right)^{-15}\right]}{\frac{0,12}{12}}$ $= R59216,72421$

OR

$$A = P(1 + i)^n$$

$$A = 192\,000 \left(1 + \frac{0,12}{12}\right)^{45}$$

$$= R300\,443,6635$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$F = \frac{4270,934 \left[\left(1 + \frac{0,12}{12}\right)^{45} - 1\right]}{\frac{0,12}{12}}$$

$$= R241\,226,9424$$

Balance on Loan

$$= R300\,443,6635 - R241\,226,9424$$

$$= R59216,7211$$

PAPER F

QUESTION 4

4.1

$$4500 = 3000 \left(1 + \frac{0,08}{12} \right)^n$$

$$\frac{3}{2} = \left(1 + \frac{0,08}{12} \right)^n$$

$$\log_{\left(1 + \frac{0,08}{12} \right)} \frac{3}{2} = n$$

$$n = 61,02 \text{ months} \quad / \quad (\text{accept } 62)$$

$$n = 5,09 \text{ years} \quad / \quad (\text{accept } 5,17)$$

NOTE: (5,08 is NOT accepted.)

4.2.1

$$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$40\,000 = \frac{x[1 - (1 + \frac{0,24}{12})^{-240}]}{\frac{0,24}{12}}$$

$$x = R806,96$$

4.2.2

$$P_o = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$P_o = \frac{806,96 \left[1 - \left(1 + \frac{0,24}{12} \right)^{-180} \right]}{\frac{0,24}{12}}$$

$$= R39205,67$$

OR

$$P_o = P(1+i)^n - \frac{x[(1+i)^n - 1]}{i}$$

$$P_o = 40\,000 \left(1 + \frac{0,24}{12} \right)^{60} - \frac{806,96 \left[\left(1 + \frac{0,24}{12} \right)^{60} - 1 \right]}{\frac{0,24}{12}}$$

$$= R39\,206,20$$

4.2.3

$$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$39205,67 = \frac{x[1 - (1 + \frac{0,18}{12})^{-180}]}{\frac{0,18}{12}}$$

$$x = R631,38$$

NOTE: If the candidate uses R39 206,20 then the answer is R631,39.

QUESTION 7

7.1

$$1 + i_{\text{eff}} = \left(1 + \frac{0,11}{2}\right)^2$$

$$i_{\text{eff}} = \left(1 + \frac{0,11}{2}\right)^2 - 1$$

$$i_{\text{eff}} = 11,30\%$$

∴ Mary has secured the better rate.

7.2.1

$$Fv = \frac{10\,000 \left[\left(1 + \frac{0,0772}{12}\right)^{114} - 1 \right]}{\frac{0,0772}{12}}$$

$$= R1\,674\,501,44$$

7.2.2	$R1\ 674\ 501,44 = \frac{30000 \left[1 - \left(1 + \frac{0,1}{12} \right)^{-n} \right]}{\frac{0,1}{12}}$ $0,46513... = \left[1 - \left(1 + \frac{0,1}{12} \right)^{-n} \right]$ $0,53486... = \left(1 + \frac{0,1}{12} \right)^{-n}$ $\log_{\left(1 + \frac{0,1}{12} \right)} 0,53486... = -n$ $n = 75,4$ She will be able to receive the money in 75 full months.
7.2.3	$Pv = \frac{30\ 000 \left[1 - \left(1 + \frac{0,1}{12} \right)^{-55} \right]}{\frac{0,1}{12}}$ $Pv = R\ 1\ 319\ 260,60$ ∴ No

DIFFERENTIAL CALCULUS

PAPER A

9.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 - \frac{1}{2}(x+h) - \left(x^2 - \frac{1}{2}x\right)}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - \frac{1}{2}x - \frac{1}{2}h - x^2 + \frac{1}{2}x}{h}$ $= \lim_{h \rightarrow 0} \frac{h\left(2x + h - \frac{1}{2}\right)}{h}$ $= 2x - \frac{1}{2}$
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9.2.1	$\frac{d}{dx} [3x^4 + \sqrt[5]{x} + a^2]$ $\frac{d}{dx} \left[3x^4 + x^{\frac{1}{5}} + a^2 \right]$ $= 12x^3 + \frac{1}{5}x^{-\frac{4}{5}}$
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9.2.2	$xy = x + x^2 - 1$ $y = 1 + x - x^{-1}$ $\frac{dy}{dx} = 1 + x^{-2}$
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QUESTION 10

10.1.1	$x^3 + 5x^2 - 8x - 12 = 0$ $(x + 1)$ is a factor $f(-1) = 0$ $(x + 1)(x^2 - 4x - 12) = 0$ $(x + 6)(x + 1)(x - 2) = 0$ $x = -6$ or $x = -1$ or $x = 2$
10.1.2	$f(x) = x^3 + 5x^2 - 8x - 12$ $f'(x) = 3x^2 + 10x - 8 = 0$ $(3x - 2)(x + 4) = 0$ $x = \frac{2}{3}$ or $x = -4$ $f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 + 5\left(\frac{2}{3}\right)^2 - 8\left(\frac{2}{3}\right) - 12 = -\frac{400}{27}$ $= -14,81$ $B\left(\frac{2}{3}; -14,81\right)$
10.1.3	$f''(x) = 6x + 10 = 0$ $x = -\frac{5}{3}$ OR $x = \frac{\frac{2}{3} + (-4)}{2} = -\frac{5}{3}$ OR $x = -\frac{b}{3a} = -\frac{5}{3}$

10.2.1	$f'(0) = -8$ $y = -8x - 12$
10.2.2	$f'(x) \cdot g'(x) > 0$ Since $g'(x) < 0$ for all $x \in R$ $(3x^2 + 10x - 8) < 0$ $(3x - 2)(x + 4) < 0$ $-4 < x < \frac{2}{3}$ OR $-4 < x < \frac{2}{3}$

PAPER B

9.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - b(x+h) - (x^2 - bx)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - bx - bh - x^2 + bx}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 - bh}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(2x + h - b)}{h}$ $f'(x) = 2x - b$
9.2.1	$\frac{d}{dx} \left[\frac{x^4}{4} - 3\sqrt[3]{x} + 7 \right]$ $\frac{d}{dx} \left[\frac{x^4}{4} - 3x^{\frac{1}{3}} + 7 \right]$ $= x^3 - x^{-\frac{2}{3}}$
9.2.2	$y = (x^{\frac{1}{3}} - 2x^{\frac{2}{3}})^2$ $y = x^{\frac{2}{3}} - 4x + 4x^{\frac{4}{3}}$ $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} - 4 + \frac{16}{3}x^{\frac{1}{3}}$

QUESTION 10

10.1	
10.1.1	$x = -5$ or $x = 1$
10.1.2	$x = -4$ or $x = 0$
10.2	$y = a(x + 5)(x - 1)$ $-15 = a(0 + 5)(0 - 1)$ $-15 = -5a$ $3 = a$ $y = 3(x + 5)(x - 1) = 3x^2 + 12x - 15$ OR $y = a(x + p)^2 + q$ $y = a(x + 2)^2 + q$ $(0; -15) : -15 = a(0 + 2)^2 + q$ $-15 = 4a + q \quad \dots (1)$ $(1; 0) : 0 = a(1 + 2)^2 + q$ $0 = 9a + q \quad \dots (2)$ $(2) - (1) : 15 = 5a \quad \therefore a = 3$ $0 = 27 + q \quad \therefore q = -27$ $y = 3(x + 2)^2 - 27$ $y = 3(x^2 + 4x + 4) - 27$ $y = 3x^2 + 12x - 15$
10.3	$f(x) = ax^3 + bx^2 + cx + d$ $f'(x) = 3ax^2 + 2bx + c$ $f'(x) = 3x^2 + 12x - 15$ Equating coefficients of equal polynomials $3a = 3$ and $2b = 12$ and $c = -15$ $a = 1$ and $b = 6$ and $c = -15$ $f(x) = x^3 + 6x^2 - 15x + d$
	$f(-3) = (-3)^3 + 6(-3)^2 - 15(-3) + d$ $f(-3) = -27 + 54 + 45 + d$ $0 = 72 + d$ $-72 = d$

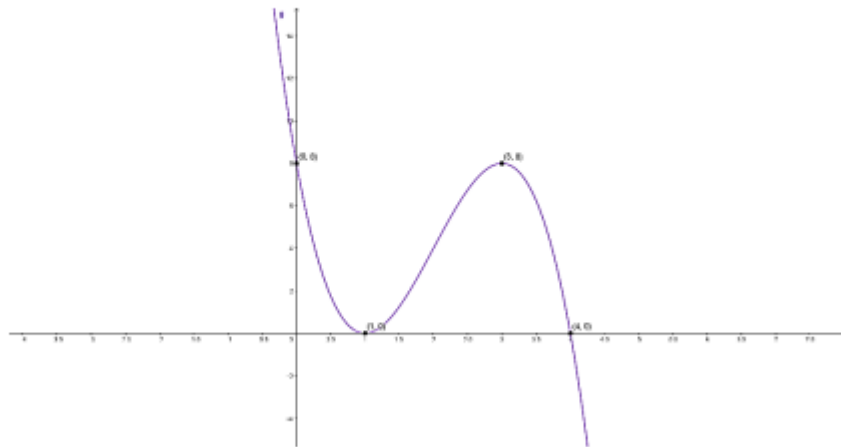
10.4	$x = -5 :$ $y = (-5)^3 + 6(-5)^2 - 15(-5) - 72 = 28$ $(-5 ; 28)$ maximum point $x = 1 :$ $y = (1)^3 + 12(1)^2 - 15(1) - 72 = -74$ $(1 ; -74)$ minimum point
10.5	$3x^2 + 12x - 15 = t$ $3x^2 + 12x - 15 - t = 0$ $\Delta = b^2 - 4ac = 0$ $\Delta = (12)^2 - 4(3)(-15 - t) = 0$ $144 + 180 + 12t = 0$ $12t = -324$ $t = -27$

PAPER C

9.1	Volume of Sphere $= \frac{4}{3}\pi(8)^3 \quad \text{or} \quad = \frac{2048\pi}{3} \quad \text{or} \quad = 2144,66$
9.2	$r^2 + x^2 = 8^2$ (Pythagoras) $r^2 = 64 - x^2$
9.3	$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(64 - x^2)(8 + x)$ $= \frac{\pi}{3}(512 + 64x - 8x^2 - x^3)$ $\frac{dV}{dx} = \frac{64\pi}{3} - \frac{16\pi}{3}x - \frac{3\pi}{3}x^2$ $0 = 64 - 16x - 3x^2$ $0 = (8 - 3x)(x + 8)$ $x = \frac{8}{3} \quad x \neq -8$ $\frac{V_{\text{cone}}}{V_{\text{sphere}}} = \frac{\frac{1}{3}\pi\left(\frac{512}{9}\right)\left(\frac{32}{3}\right)}{\frac{2048\pi}{3}}$ $= \frac{8}{27} = 0,3$

QUESTION 10

10.1



10.2

$$f(x) = a(x-1)^2(x-4)$$

$$8 = a(3-1)^2(3-4)$$

$$8 = a(-4)$$

$$a = -2$$

$$f(x) = -2(x-1)^2(x-4)$$

$$f(x) = -2(x^2 - 2x + 1)(x-4)$$

$$f(x) = -2x^3 + 12x^2 - 18x + 8$$

10.3

$$f(x) = -2x^3 + 12x^2 - 18x + 8$$

$$f'(x) = -6x^2 + 24x - 18$$

$$f''(x) = -12x + 24$$

$$f''(x) < 0$$

$$-12x + 24 < 0$$

$$-x < -2$$

$$x > 2$$

QUESTION 12

12.1	$h'(x) = -3x^2 + 2ax + b$ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ $0 = -3 - 2a + b$ $2a - b = -3 \quad \dots (i)$ $h'(2) = -3(2)^2 + 2a(2) + b$ $0 = -12 + 4a + b$ $4a + b = 12 \quad \dots (ii)$ $(ii) + (i): \quad 6a = 9$ $a = \frac{3}{2}$ $\therefore 2\left(\frac{3}{2}\right) - b = -3$ $b = 6$
12.2	Average Gradient $= \frac{10 - (-3,5)}{2 - (-1)}$ $= \frac{13,5}{3}$ $= \frac{9}{2}$

12.3	$h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2 ; 2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$
12.4	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$ OR $x = \frac{-1 + 2}{2}$ $x = \frac{1}{2}$
12.5	$p > 3,5 \text{ or } p < -10$

PAPER D

QUESTION 8

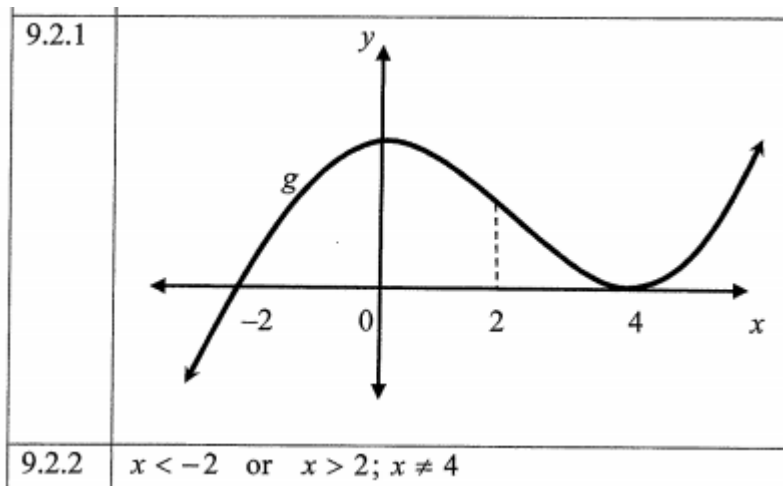
8.1	$f(x) = 2x^2 - 5x + 3$ $f(x + h) = 2(x + h)^2 - 5(x + h) + 3$ $= 2(x^2 + 2xh + h^2) - 5x - 5h + 3$ $= 2x^2 + 4xh + 2h^2 - 5x - 5h + 3$ $f(x + h) - f(x) = 2x^2 + 4xh + 2h^2 - 5x - 5h + 3$ $- (2x^2 - 5x + 3)$ $= 4xh + 2h^2 - 5h$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 5h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(4x + 2h - 5)}{h}$ $= \lim_{h \rightarrow 0} (4x + 2h - 5)$ $= 4x - 5$
-----	--

8.2	$y = \frac{2x^2}{3\sqrt{x}} - \frac{2x^3 + 1}{x^3}$ $= \frac{2x^2}{3x^{\frac{1}{2}}} - 2 - \frac{1}{x^3}$ $= \frac{2}{3}x^{\frac{3}{2}} - 2 - x^{-3}$ $\frac{dy}{dx} = x^{\frac{1}{2}} + 3x^{-4}$
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QUESTION 9

9.1.1	$f(x) = -2x^3 + 5x^2 + 4x - 3$ $0 = (x - 3)(-2x^2 - x + 1)$ $x - 3 = 0 \quad \text{or} \quad -2x^2 - x + 1 = 0$ $x = 3 \qquad 2x^2 + x - 1 = 0$ $(3; 0) \qquad (2x - 1)(x + 1) = 0$ $2x = 1 \quad \text{or} \quad x = -1$ $x = \frac{1}{2} \qquad (-1; 0)$ $\left(\frac{1}{2}; 0\right)$
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9.1.2	$f'(x) = -6x^2 + 10x + 4$ $0 = -6x^2 + 10x + 4$ $3x^2 - 5x - 2 = 0$ $(3x + 1)(x - 2) = 0 \quad \text{or} \quad x = \frac{-10 \pm \sqrt{10^2 - 4(-6)(4)}}{2(-6)}$ $3x = -1 \quad \text{or} \quad x = 2$ $x = -\frac{1}{3}$
9.1.3	$f''(x) = -12x + 10$ $0 = -12x + 10$ $12x = 10$ $x = \frac{10}{12} = \frac{5}{6}$ $\therefore x < \frac{5}{6}$



QUESTION 10

10.1	$N(t) = t^3 - 12t^2 + 36t + 8$ $N(0) = 8$ $\therefore 8$ people
10.2	$N'(t) = 3t^2 - 24t + 36$ increasing $N'(t) \geq 0$ $3t^2 - 24t + 36 \geq 0$ $t^2 - 8t + 12 \geq 0$ $(t - 6)(t - 2) \geq 0$ $t \geq 6$ or $t \leq 2$ $\therefore$ for first 2 hours after opening or 6 hours after opening until closing time
10.3	Minimum turning point at $t = 6$ hours after opening

QUESTION 11

11.1	$Area = x^2 + 3x^2 + 4xh$ $Area = 4x^2 + 4xh$ $V = x^2h = 1000$ $h = \frac{1000}{x^2}$ $A = 4x^2 + 4x\left(\frac{1000}{x^2}\right)$ $A = 4x^2 + \frac{4000}{x}$
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11.2	$A = 4x^2 + 4000x^{-1}$ $A' = 8x - 4000x^{-2} = 0$ $8x = \frac{4000}{x^2}$ $x^3 = 500$ $x = \sqrt[3]{500} = 7,94 \text{ cm}$ $h = \frac{1000}{(7,94)^2} = 15,86 \text{ cm}$
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PAPER E

QUESTION/VRAAG 8

8.1	$f(x+h) = 3 - 2(x+h)^2$ $= 3 - 2x^2 - 4xh - 2h^2$ $f(x+h) - f(x) = 3 - 2x^2 - 4xh - 2h^2 - 3 + 2x^2$ $= -4xh - 2h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= \lim_{h \rightarrow 0} (-4x - 2h)$ $= -4x$
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$$\begin{aligned}
 8.2 \quad y &= \frac{12x^2 + 2x + 1}{6x} \\
 &= 2x + \frac{1}{3} + \frac{1}{6x} \\
 &= 2x + \frac{1}{3} + \frac{1}{6}x^{-1} \\
 \frac{dy}{dx} &= 2 - \frac{1}{6}x^{-2} \\
 &= 2 - \frac{1}{6x^2}
 \end{aligned}$$

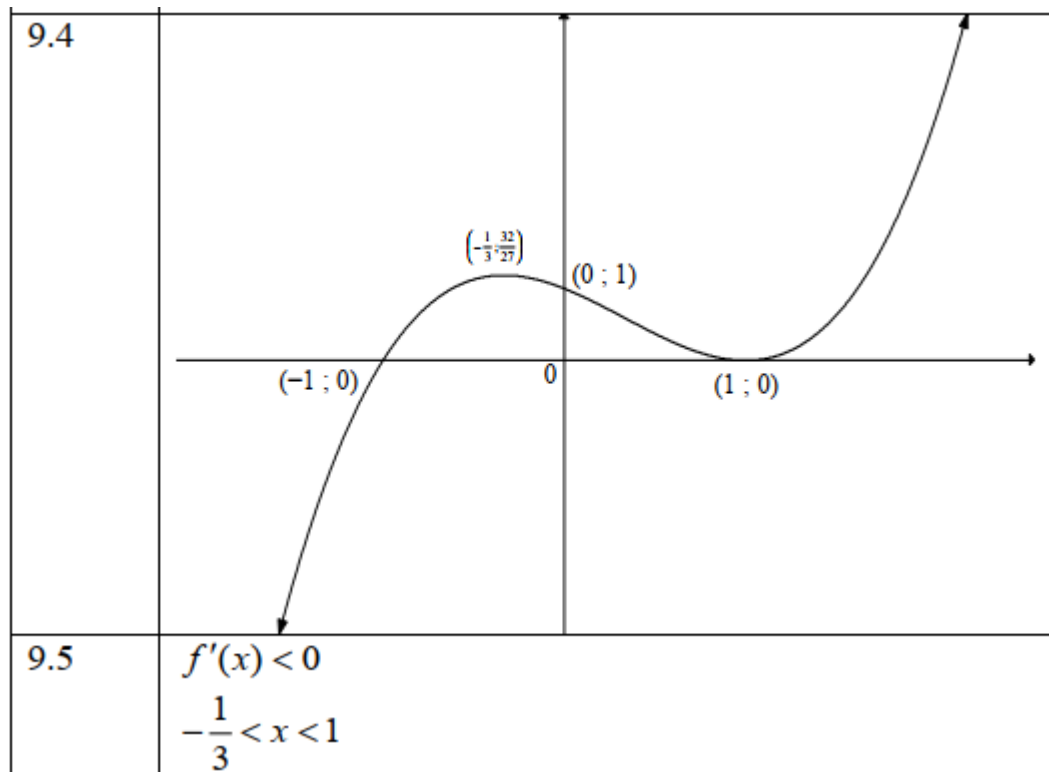
$$\begin{aligned}
 8.3 \quad y &= x^3 + bx^2 + cx - 4 \\
 y' &= 3x^2 + 2bx + c \\
 y'' &= 6x + 2b \\
 \text{At point of inflection:} \\
 y'' &= 6x + 2b = 0 \\
 \text{Substitute } x = 2: \\
 6(2) + 2b &= 0 \\
 2b &= -12 \\
 b &= -6 \\
 y &= x^3 - 6x^2 + cx - 4 \\
 \text{Substitute } (2; 4): \\
 4 &= 2^3 - 6(2)^2 + c(2) - 4 \\
 2c &= 24 \\
 c &= 12 \\
 y &= x^3 - 6x^2 + 12x - 4
 \end{aligned}$$

QUESTION/VRAAG 9

9.1	(0 ; 1)
9.2	$f(x) = x^3 - x^2 - x + 1$ $f(x) = x^2(x-1) - (x-1)$ $f(x) = (x-1)(x^2 - 1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (-1; 0); (1; 0)

OR

	$f(x) = x^3 - x^2 - x + 1$ $f(x) = (x-1)(x^2 - 1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (-1; 0); (1; 0)
9.3	$f(x) = x^3 - x^2 - x + 1$ $f'(x) = 3x^2 - 2x - 1$ $f'(x) = 0$ $(3x+1)(x-1) = 0$ $x = -\frac{1}{3} \quad \text{or} \quad x = 1$ $y = \frac{32}{27} \quad y = 0$ $\left(-\frac{1}{3}; \frac{32}{27}\right) \quad (1; 0)$



QUESTION 10

10.1	$s(t) = 2t^2 - 18t + 45$ $s'(t) = 4t - 18$ $s'(0) = 4(0) - 18$ $= -18 \text{ m/s}$
10.2	$s''(t) = 4 \text{ m/s}^2$

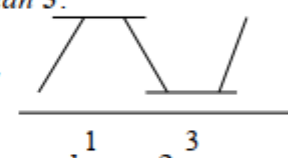
10.2	$s''(t) = 4\text{m/s}^2$
10.3	$4t - 18 = 0$ $4t = 18$ $t = \frac{9}{2}\text{seconds or } 4,5\text{seconds}$ OR $s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$ $t = \frac{9}{2}\text{seconds or } 4,5\text{seconds}$ OR $s(t) = 2t^2 - 18t + 45$ $t = -\frac{-18}{2(2)}$ $t = \frac{9}{2}\text{seconds or } 4,5\text{seconds}$

PAPER F

8.1	$f(x+h) = \frac{4}{x+h}$ $f(x+h) - f(x) = \frac{4}{x+h} - \frac{4}{x}$ $= \frac{4x - 4(x+h)}{x(x+h)}$ $= \frac{4x - 4x - 4h}{x(x+h)}$ $= \frac{-4h}{x(x+h)}$ $\frac{f(x+h) - f(x)}{h} = \frac{\frac{-4h}{x(x+h)}}{h}$ $= \frac{-4h}{xh(x+h)}$ $= \frac{-4}{x(x+h)}$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-4}{x(x+h)}$ $= \frac{-4}{x^2}$
8.2.1	$y = 5x^2 + 5x + 2$ $\frac{dy}{dx} = 10x + 5$
8.2.2	$D_x \left[\sqrt[3]{x^2} - \frac{1}{2}x \right]$ $= D_x \left[x^{\frac{2}{3}} - \frac{1}{2}x \right]$ $= \frac{2}{3}x^{\frac{-1}{3}} - \frac{1}{2}$

8.3	$p(x) = x^3 + 2x$ $p'(x) = 3x^2 + 2$ $3x^2 \geq 0$ or / of $x^2 \geq 0$ for all/vir alle $x \in \mathbf{R}$ $\therefore 3x^2 + 2 \geq 2 > 0$ for all/vir alle $x \in \mathbf{R}$ i.e. $p'(x) > 0$ for all/vir alle $x \in \mathbf{R}$ i.e. all tangents to p have gradient greater than (or equal to) 2. Thus there is no tangent to p that has negative gradient.
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QUESTION/VRAAG 9

9.1	$x = 1$ or $x = 3$
9.2	$1 < x < 3$
9.3	For a point x close to 3/Vir 'n punt naby aan 3: If $x < 3$, $f'(x) < 0 \Rightarrow f$ decreasing/dalend If $x > 3$, $f'(x) > 0 \Rightarrow f$ increasing/stygend Therefore: f has a local minimum at/fhet lokale minimum by $x = 3$  OR/OF At $x = 3$, the gradient function changes from negative to positive therefore the function will have a local minimum point at $x = 3$/ By $x = 3$ verander die gradiëntfunksie van negatief na positief dus sal die funksie 'n lokale minimum punt hê by $x = 3$.
	OR/OF $f'(3) = 0$ and $f''(3) > 0$ therefore the function will have a local minimum point at $x = 3$ / $f''(3) > 0$ dus sal die funksie 'n lokale minimum punt hê by $x = 3$.
9.4	$f''(x) = 0$ at the turning point of/by die draaipunt van $f'(x)$ Using symmetry/Deur simmetrie $x = \frac{1+3}{2}$ $= 2$
9.5	Concave up if/Konkaaf op as $f''(x) > 0$ $x > 2$

QUESTION/VRAAG 10

	Given: $M(t) = t^3 - 9t^2 + 3000$; $0 \leq t \leq 30$
10.1	$M(0) = 0^3 - 9(0)^2 + 3000$ $= 3000g$ or $3kg$
10.2	$t^3 - 9t^2 + 3000 = 3000$ $t^3 - 9t^2 = 0$ $t^2(t - 9) = 0$ $t = 0$ or $t = 9$ Baby's mass will return to the birth mass on the 9 th day/ <i>Baba se massa keer terug na massa by geboorte op die 9^{de} dag.</i>
10.3	$M'(t) = 0$ $3t^2 - 18t = 0$ $3t(t - 6) = 0$ $t = 0$ or $t = 6$ Baby's mass will be a minimum on the 6 th day/ <i>Baba se massa sal 'n minimum wees op die 6^{de} dag.</i>
10.4	$M'(t) = 3t^2 - 18t$ $M''(t) = 6t - 18$ $0 = 6t - 18$ $t = 3$
	OR / OF Using symmetry/ <i>Deur simmetrie</i> : $t = \frac{0+6}{2}$ $= 3$

PROBABILITY AND COUNTING

BASIC PROBABILITY QUESTIONS AND/OR VENN DIAGRAMS

PAPER A

QUESTION 4

4.1	$P(A \text{ or } B) = 0,3 + 0,5$ $= 0,8$
4.2	Since A and B are independent $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= 0,3 + 0,5 - 0,15$ $= 0,65$

PAPER B

QUESTION 4

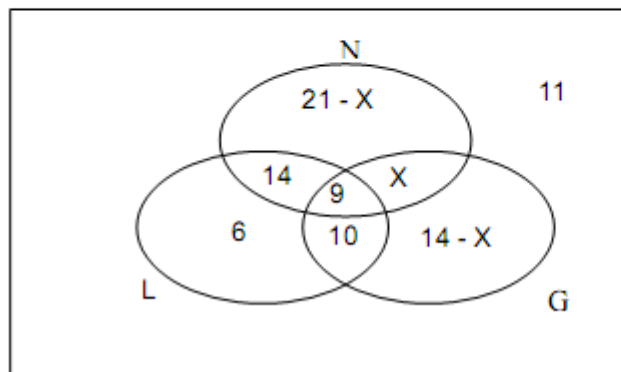
4.1	Since A and C are mutually exclusive, there is no intersection of A and C $\therefore P(A \text{ and } C) = 0$.
4.2	Since B and C are independent, $P(B \text{ and } C) = P(B) \cdot P(C)$. $P(B \text{ and } C) = (0,4)(0,2) = 0,08$
4.3	Since A and B are independent, $P(A \text{ and } B) = P(A) \cdot P(B)$. $P(A \text{ and } B) = (0,3)(0,4) = 0,12$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= 0,3 + 0,4 - 0,12$ $= 0,58$

PAPER D

QUESTION 4

4.1.1 11 students

4.1.2 Let N represent students reading the *National Geographic* magazine, G represent students reading the *Getaway* magazine and L represent students reading the *Leadership* magazine.



$$\begin{aligned}
 4.1.3 \quad 21 - x + x + 14 - x + 9 + 14 + 10 + 6 + 11 &= 80 \\
 85 - x &= 80 \\
 x &= 5
 \end{aligned}$$

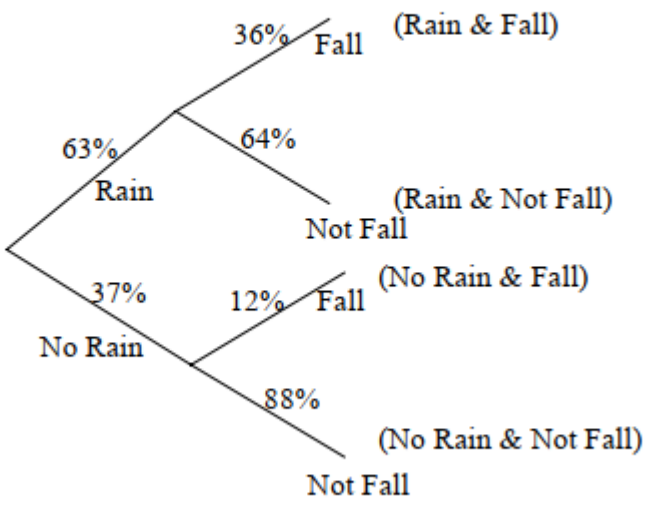
$$4.1.4 \quad P(\text{student reads at least two magazines}) = \frac{5 + 14 + 10 + 9}{80} = 0,475$$

$$\begin{aligned}
 4.2.1 \quad &P(\text{smoke detected by device A or device B}) \\
 &= P(\text{smoke detected by A}) + P(\text{smoke detected by B}) - P(\text{smoke detected by both}) \\
 &= 0,95 + 0,98 - 0,94 \\
 &= 0,99
 \end{aligned}$$

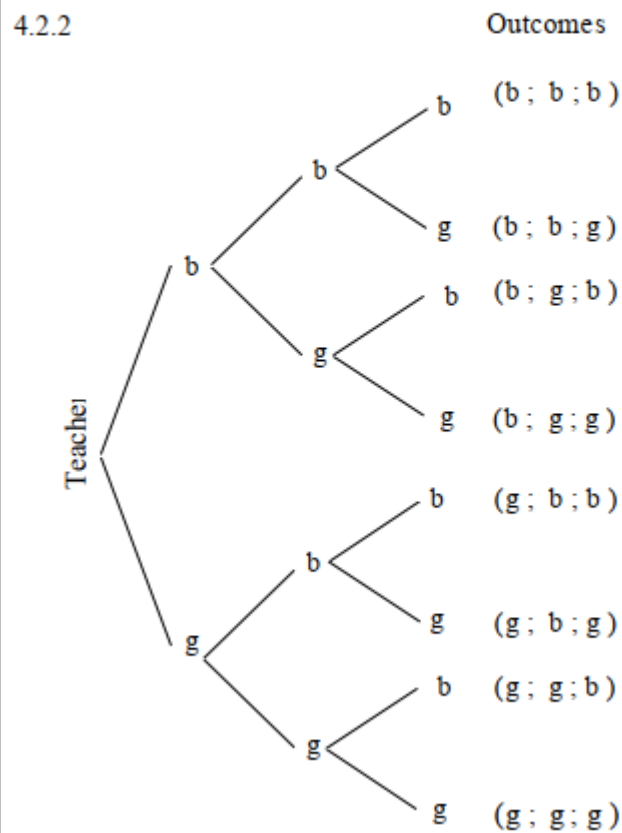
$$4.2.2 \quad P(\text{smoke not detected}) = 1 - 0,99 = 0,01$$

PAPER E

QUESTION 3

3.1	
3.2	$ \begin{aligned} P(\text{Not Fall}) &= \left(\frac{37}{100} \times \frac{88}{100} \right) + \left(\frac{63}{100} \times \frac{64}{100} \right) \\ &= \frac{407}{1250} + \frac{252}{625} \\ &= \frac{911}{1250} \\ &= 0,7288 \end{aligned} $
3.3	$ \begin{aligned} P(\text{Dry & Fall}) &= \frac{37}{100} \times \frac{12}{100} \\ &= \frac{111}{2500} \\ &= 0,0444 \end{aligned} $

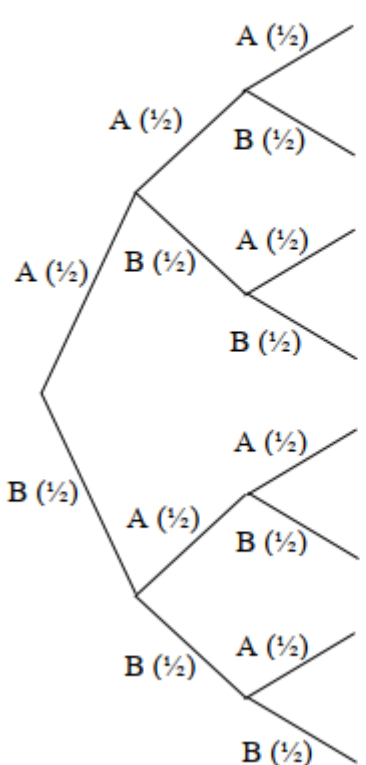
4.2.1 $P(\text{boy chosen first}) = \frac{20}{35} = \frac{4}{7} = 0,57.$



4.2.3 $P(b ; g ; b) = \frac{20}{35} \times \frac{15}{34} \times \frac{19}{33} = \frac{190}{1309} = 0,15$

4.2.4 $P(g ; g ; g) = \frac{15}{35} \times \frac{14}{34} \times \frac{13}{33} = \frac{13}{187} = 0,07$

4.2.5 $P(\text{at least one boy}) = 1 - P(\text{three girls chosen})$
 $= 1 - 0,07$
 $= 0,93$

5.1	Let A represent Alfred winning a point and B represent Barry winning a point. Outcomes 
5.2	$P(\text{Barry wins three points}) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$
5.3	$ \begin{aligned} &P(\text{Alfred wins two points and Barry wins one point}) \\ &= P(A; A; B) + P(A; B; A) + P(B; A; A) \\ &= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{3}{8} \end{aligned} $
5.4	$ \begin{aligned} &P(\text{Alfred wins 3 of the four points}) \\ &= P(AAAB) + P(AABA) + P(ABAA) + P(BAAA) \\ &= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\ &= 4\left(\frac{1}{2}\right)^4 \\ &= \frac{1}{4} \end{aligned} $

CONTINGENCY TABLES

PAPER A

5.1.1	$P(\text{male}) = \frac{120}{236}$ $= \frac{30}{59}$ $= 0,51 (0,508474\dots)$
5.1.2	$P(\text{female and plays sport})$ $= \frac{67}{236}$ $= 0,28 (0,2838983051\dots)$
5.2	No. From the table, $P(\text{male and do not play sport}) = \frac{51}{236}$, which is greater than zero. Since the probability of the intersection of these two events is greater than zero, these events are not mutually exclusive.

5.3	$P(\text{male}) = \frac{120}{236}$ $P(NS) = \frac{100}{236}$ $P(\text{male}) \times P(NS) = \frac{120}{236} \times \frac{100}{236}$ $= \frac{750}{3\,481}$ $= 0,22 \quad (0,215455\dots)$ $P(\text{male and NS}) = \frac{51}{236}$ $= 0,22 \quad (0,2161016949\dots)$ So, $P(\text{male}) \times P(NS) \neq P(\text{male and NS})$ Therefore the events 'male' and 'do not play sport' are independent (correct to TWO decimal places). OR The events are not independent as there is a discrepancy from the third decimal place.
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PAPER B

5.1	$a = 450$ $b = 319$ $c = 298$ $d = 748$
5.2	$P(\text{Female who has not broken a limb})$ $= \frac{298}{1530}$ $= \frac{149}{765}$
5.3	$P(\text{Female \& broken a limb})$ $= \frac{450}{1530}$ $= \frac{5}{17}$ $= 0,2941176471...$ $= 0,29$ $P(\text{Female}) \times P(\text{Broken a limb})$ $= \frac{748}{1530} \times \frac{913}{1530}$ $= 0,29$ The events of being female and having broken a limb are independent. If a candidate answers not independent due to the fact that the answers are not accurate to more than 2 decimal places, award full marks.

PROBABILITY AND COUNTING PRINCIPLE

PAPER A

10.2.1	$5!$ $= 120$
10.2.2	5^5 $= 3125$
10.3	$n(E) = 5! \times 2! \times 2!$ $n(S) = 7!$ $P(E) = \frac{5! \times 2! \times 2!}{7!}$ $= \frac{2}{21}$

PAPER B

QUESTION 7

7.1	Number of ways $= 8 \times 8$ $= 64$
7.2	Number of ways for a 4-digit number $= 8 \times 7 \times 6 \times 5$ $= 1\,680$ OR Number of ways for a 4-digit number $= \frac{8!}{(8-4)!}$ $= \frac{8!}{4!}$ $= 1680$
7.3	Numbers between 4 000 and 5 000 $= 1 \times 8 \times 8 \times 8$ $= 512$

PAPER C

QUESTION/VRAG 12

12.1.1	$26 \times 25 \times 24 \times 23 \times 22$ $= 7\,893\,600$
12.1.2	$24 \times 23 \times 22$ $= 12\,144$

PAPER D

5.1.1	Number of PIN codes $= 10 \times 10 \times 10 \times 10 \times 10$ $= 10^5$ $= 100\,000$
5.1.2	Number of PIN codes $= 10 \times 9 \times 8 \times 7 \times 6$ $= 30\,240$ OR Number of PIN codes $= \frac{10!}{5!}$ $= 30\,240$
5.2	Number of PINs that DO NOT contain 9s $= 9 \times 9 \times 9 \times 9 \times 9$ $= 59\,049$ P(at least one 9) $= 1 - \text{P(no 9s)}$ $= 1 - \frac{59\,049}{100\,000}$ $= 0,41$

PAPER E

5.2.1	Any learner seated in any position in: $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$ $= 720$ different ways.
5.2.2	$2 \times 5! = 240$

PAPER F

QUESTION 6

6.1	Number of different ways the shirts and trousers can be arranged $= (7 + 4)!$ $= 11!$ $= 39\,916\,800$
6.2	Number of ways so that the shirts are together and trousers are together $= 7! \cdot 4! \cdot 2$ $= 241\,920$
6.3	P(Shirt at beginning and trouser at the end) $= \frac{9! \times 4 \times 7}{11!}$ $= \frac{14}{55}$

PAPER G

QUESTION 5

5.1	Number of arrangements $= 7!$ $= 5040$
5.2	Number of arrangements $= 5!$ $= 120$
5.3	Number of arrangements $= 3! \times 5!$ $= 720$

PAPER H

QUESTION/VRAG 11

11.1	$8 \times 7 \times 6 \times 5 \times 4$ or $\frac{8!}{3!}$ $= 6720$
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PAPER 2

STATISTICS

PAPER A

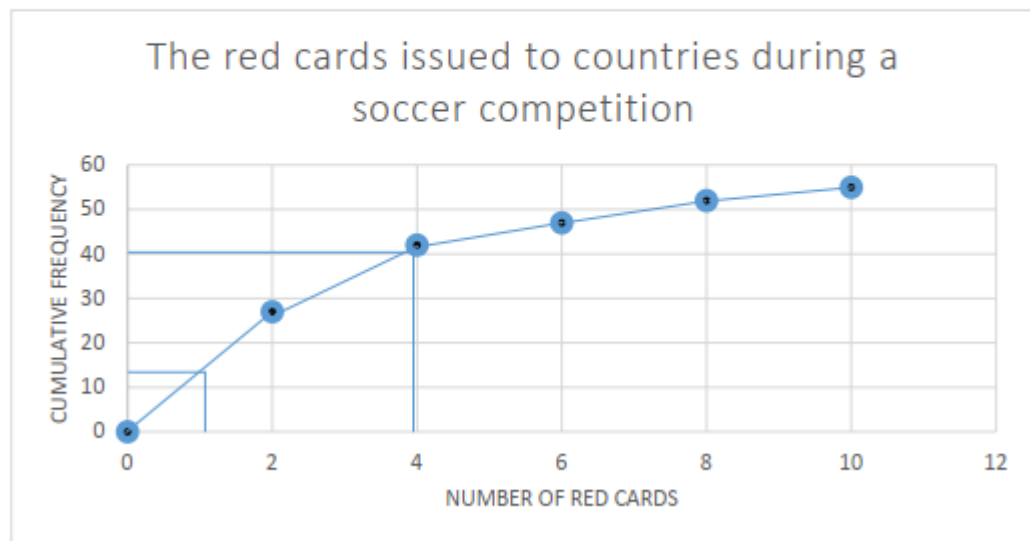
QUESTION 1

NUMBER OF RED CARDS	NUMBER OF COUNTRIES (<i>f</i>)	MIDPOINT OF INTERVAL (<i>x</i>)	<i>f</i> · <i>x</i>
$0 < x \leq 2$	27	1	27
$2 < x \leq 4$	15	3	45
$4 < x \leq 6$	5	5	25
$6 < x \leq 8$	5	7	35
$8 < x \leq 10$	3	9	27
TOTAL	55		159

1.1 Estimated mean = $\frac{159}{55} = 2,89 \approx 3$ red cards

Answer only full marks

1.2



1.3 $Q_3 = 4$ and $Q_1 = 1 \therefore IQR = 4 - 1 = 3$ red cards

QUESTION 2

2.1	$A = 5,97$; $B = 2,18$ $Y = 5,97 + 2,18x$ Answer only full marks
2.2	Estimated monthly income $y = 5,97 + 2,18(9)$ $= 25,59$ $\therefore$ Monthly income = R25598,89 If 9000 is used only 1 mark
2.3	$r = 0,94$
2.4	Very strong positive relationship between the monthly rent and the monthly income.

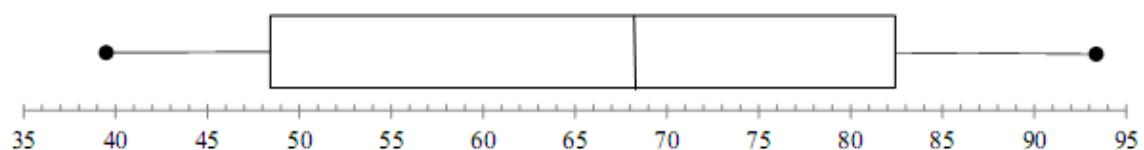
PAPER B

QUESTION 1

39	42	48	54	62	68	78	78	82	91	93
----	----	----	----	----	----	----	----	----	----	----

1.1	$\bar{x} = 66,82$ or 66,82 thousands
1.2	$\sigma = 18,3$
1.3	$(\bar{x} - \sigma; \bar{x} + \sigma) = (48,52 ; 85,12)$ $\therefore$ 6 countries

1.4



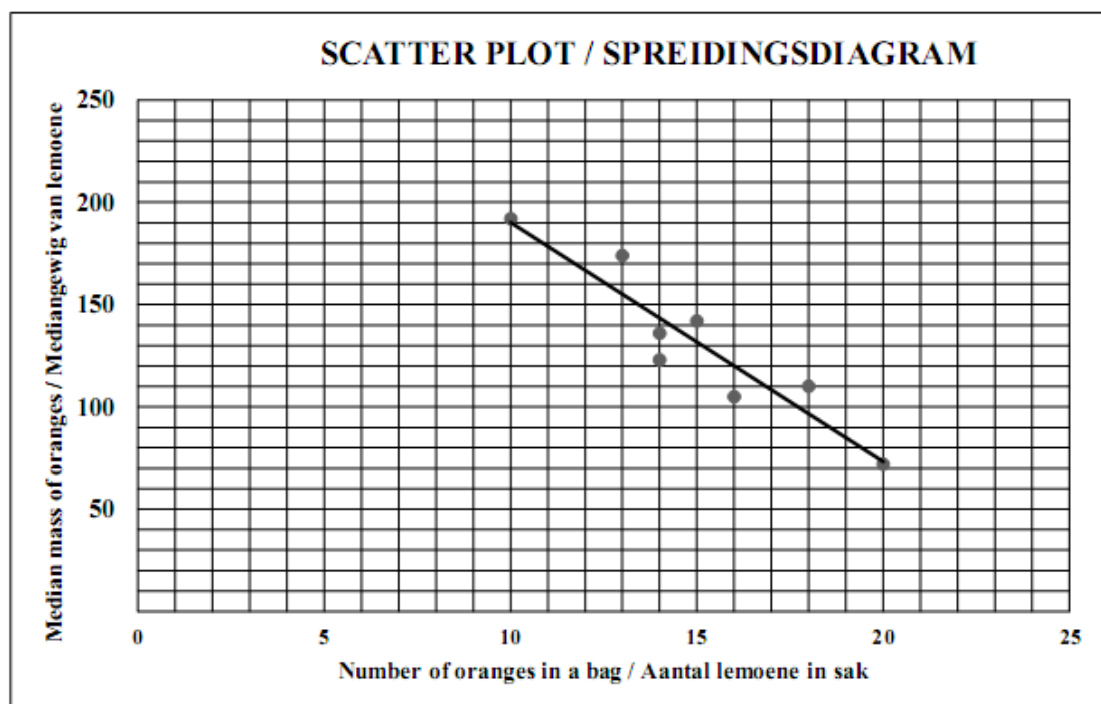
1.5	The data is skewed to the left/negatively skewed
1.6.1	The sum of the data provided for the years must increase.
1.6.2	IQR will remain the same or change.

QUESTION 2

YEAR	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Agriculture (x)	4,2	3,4	3,1	2,7	2,9	3,0	2,9	3,0	2,6	2,5	2,6
Mining (y)	19,2	19,4	19,2	18,5	17,5	17,0	16,8	15,2	14,2	12,8	12,4

2.1	$a = 5,65$ and $b = 3,65$ $y = 5,65 + 3,65x$
2.2	$y = 5,65 + 3,65x$ $= 5,65 + 3,65(1,2)$ $= 10,03\%$
2.3	strong positive correlation of the data $r = 0,7$

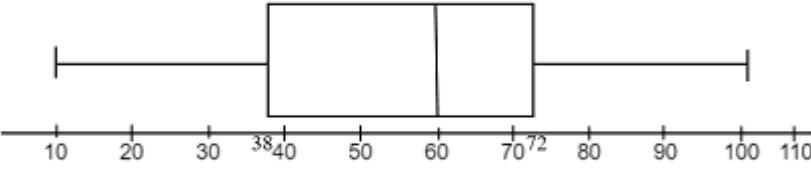
PAPER C



1.1	$a = 307,20$ $b = -11,70$ $\hat{y} = 307,20 - 11,7x$
1.2	$r = -0,93$

1.3	See scatter plot above/ <i>sien spreidingsdiagram hierbo</i> (10 ; 190,2) (20 ; 73,2)
1.4	Negative strong association / <i>Negatiewe sterk assosiasie</i>
1.5	$\hat{y} = 307,20 - 11,7(12)$ $= 166,8$

QUESTION / VRAAG 2

2.1.1	100
2.1.2	Median / <i>Mediaan</i> = 62
2.1.3	
2.1.4	Skewed to the left / <i>Skeef na links</i>
2.2	$b = 20$ $\frac{d-a}{2} = 8$ $2a = d$ $\text{sub } \frac{2a-a}{2} = 8$ $a = 16$ $d = 32$ $5 + 16 + 19 + 20 + c + 32 + 35 = 7 \times 22$ $\therefore c = 27$

PAPER D

QUESTION 1

1.1	$\bar{x} = \frac{1\,581}{31}$ $= 51$ OR / OF $\bar{x} = 51 \text{ (calculator method / sakrekenaar metode)}$
1.2	∴ skewed to the left
1.3	Physical Sciences performed better. Q_1 is 40% in Physical Sciences and 28% in Mathematics which indicates the lower 25% of the class performed much better in Physical Sciences than in Mathematics. <i>Fisiese Wetenskappe presteer beter.</i> Q_1 is 40% in Fisiese Wetenskappe en 28% in Wiskunde wat aandui dat die onderste 25% van die klas heelwat beter presteer in Fisiese Wetenskappe as in Wiskunde.
1.4	Accept any mark between 40 – 50. <i>Aanvaar enige punt tussen 40 – 50 .</i>
1.5	The greatest difference is $87\% - 71\% = 16\%$

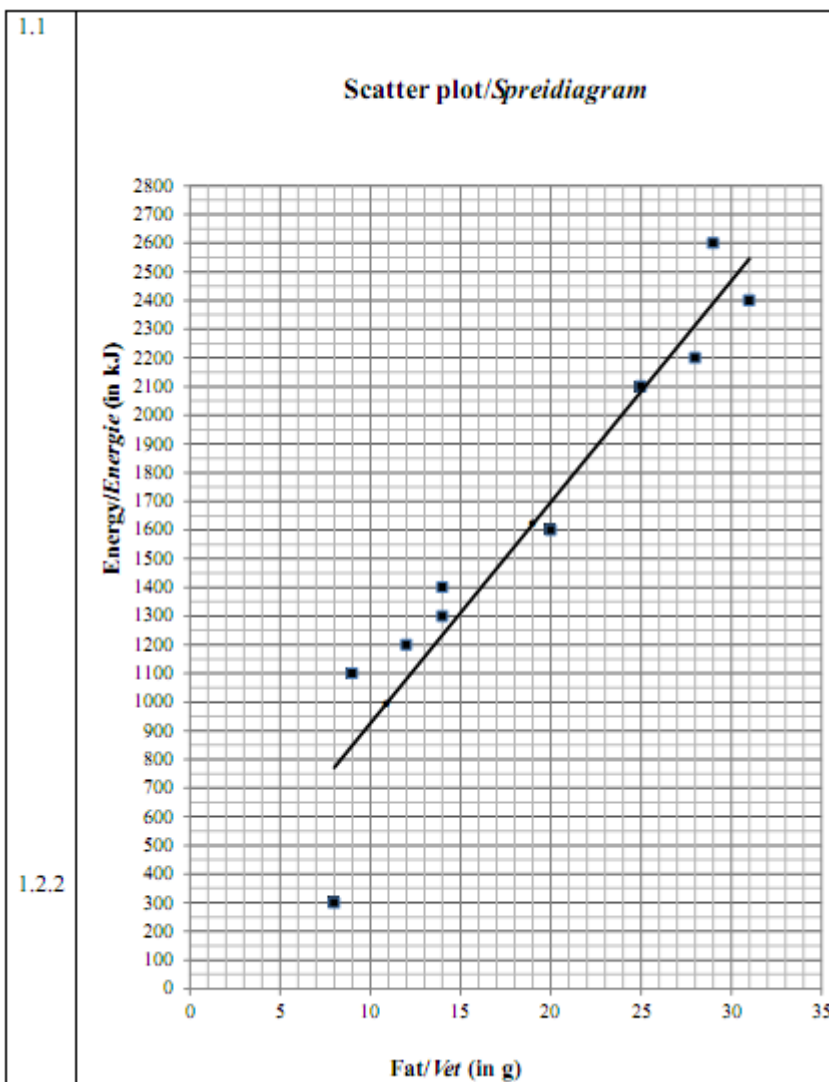
QUESTION / VRAAG 2

2.1	$a = 12,41$ $b = 0,49$ $\hat{y} = 12,41 + 0,49x$
2.2	$\hat{y} = 12,41 + 0,49x$ $= 12,41 + 0,49(150)$ $= 85,91 \approx 86\%$ OR/OF $\hat{y} = 85,17$
2.3	$\hat{y} = 12,41 + 0,49x$ The y-intercept is 12,41 which means that a learner who did not begin the exam achieved 12,41%. This is clearly impossible.

2.4	10,28
2.5	$63,9 - \sigma = p$ $63,9 + \sigma = 103,59$ $127,92 = p + 103,59$ $p = 24,33$ OR / OF $\sigma = 103,59 - 63,96$ $= 39,63$ $p = 63,96 - 39,63$ $= 24,33$

PAPER E

Question 1



1.2.1	$\hat{y} = 154,60 + 77,13(18)$ $= 1\,542,94 \approx 1\,500 \text{ kJ}$
1.3	(8 ; 300)
1.4	$r = 0,9520... \approx 0,95$
1.5	very strong positive relationship/

Question 2

2.1	mean/ <i>gemiddelde</i> = $\frac{2(0) + 3(3) + 4(2) + ...12(1)}{30} = \frac{202}{30}$ $= 6,73$
2.2	median/ <i>mediaan</i> = $\frac{T_{15} + T_{16}}{2} = \frac{7 + 7}{2} = 7$
2.3	SD/ <i>SA</i> = $2,264... \approx 2,26$
2.4	$(6,73 - 2,26 ; 6,73 + 2,26)$ $= (4,47 ; 8,99)$ $\therefore 4 + 4 + 8 + 3 = 19 \text{ times/keer}$

PAPER F

QUESTION/VRAAG 2

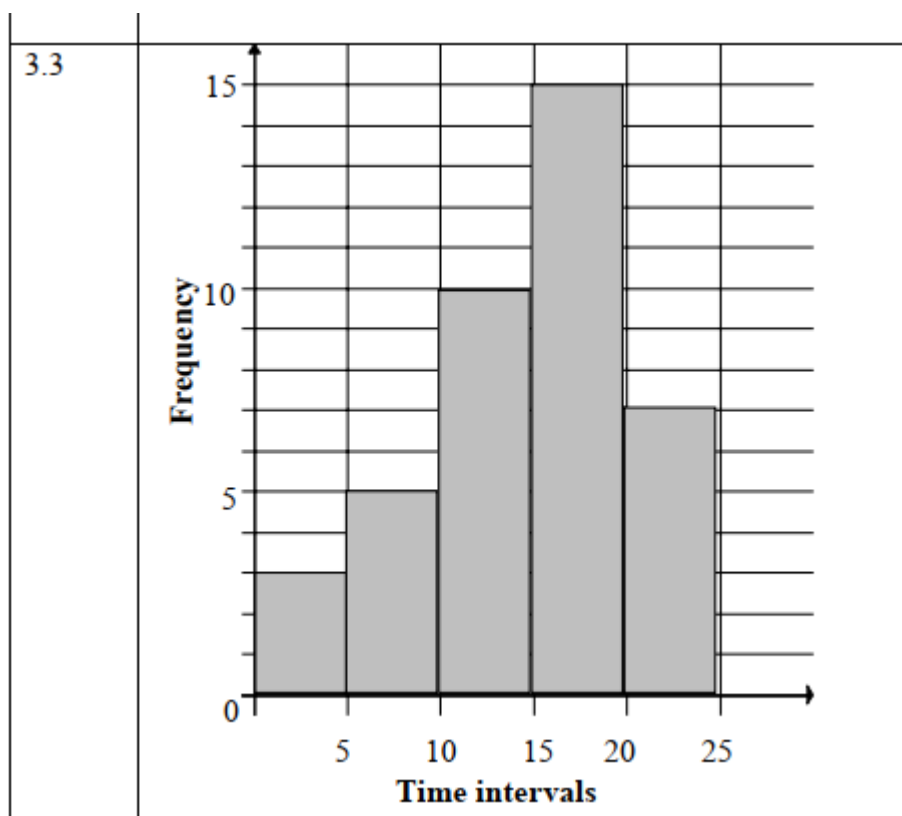
2.1	Positively skewed OR skewed to the right/ <i>positief skeef OF skeef na regs</i>												
2.2	Range/ <i>Omvang</i> = $2,21 - 1,39 = 0,82 \text{ m}$												
2.3	<table border="1"> <thead> <tr> <th>Intervals <i>Klasse</i></th><th>Cumulative frequency <i>Kumulatiewe frekwensie</i></th></tr> </thead> <tbody> <tr> <td>$1,3 \leq x < 1,5$</td><td>24</td></tr> <tr> <td>$1,5 \leq x < 1,7$</td><td>95</td></tr> <tr> <td>$1,7 \leq x < 1,9$</td><td>133</td></tr> <tr> <td>$1,9 \leq x < 2,1$</td><td>156</td></tr> <tr> <td>$2,1 \leq x < 2,3$</td><td>160</td></tr> </tbody> </table>	Intervals <i>Klasse</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$1,3 \leq x < 1,5$	24	$1,5 \leq x < 1,7$	95	$1,7 \leq x < 1,9$	133	$1,9 \leq x < 2,1$	156	$2,1 \leq x < 2,3$	160
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$1,7 \leq x < 1,9$	133												
$1,9 \leq x < 2,1$	156												
$2,1 \leq x < 2,3$	160												

2.4	OGIVE/OGIEF
2.5	method (using 80 to determine the height) 1,65 (accept any value between 1,6 and 1,69)
2.6.1	The mean would change by 0,1 m <i>Die gemiddelde sal met 0,1 m verander</i>
2.6.2	No influence/change as there is no difference in variation of data./<i>Geen invloed</i>

PAPER G

QUESTION 3

3.1	40												
3.2	<table> <tr> <th>Time, t, in minutes</th><th>Frequency</th></tr> <tr> <td>$0 \leq t < 5$</td><td>3</td></tr> <tr> <td>$5 \leq t < 10$</td><td>5</td></tr> <tr> <td>$10 \leq t < 15$</td><td>10</td></tr> <tr> <td>$15 \leq t < 20$</td><td>15</td></tr> <tr> <td>$20 \leq t < 25$</td><td>7</td></tr> </table>	Time, t , in minutes	Frequency	$0 \leq t < 5$	3	$5 \leq t < 10$	5	$10 \leq t < 15$	10	$15 \leq t < 20$	15	$20 \leq t < 25$	7
Time, t , in minutes	Frequency												
$0 \leq t < 5$	3												
$5 \leq t < 10$	5												
$10 \leq t < 15$	10												
$15 \leq t < 20$	15												
$20 \leq t < 25$	7												



ANALYTICAL GEOMETRY

PAPER A

QUESTION 3

3.1.1	$m_{LM} = \frac{0-1}{4-1} = -\frac{1}{3}$ $m_{MN} = \frac{2-0}{8-4} = \frac{1}{2}$
3.1.2	$KM = \sqrt{(4-4)^2 + (10-0)^2}$ $= \sqrt{100}$ $= 10 \text{ units}$ Answer only full marks
3.1.3	$m_{MN} = \frac{1}{2}$ $\tan \theta = \frac{1}{2}$ $\theta = 26,57^\circ$
3.1.4	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $\left(\frac{1+8}{2}, \frac{1+2}{2} \right)$ $\left(\frac{9}{2}, \frac{3}{2} \right)$
3.2	$m_{KL} = \frac{10-1}{4-1} = 3$ $m_{KL} \times m_{LM} = 3 \times \left(-\frac{1}{3} \right)$ $= -1$ $\therefore KL \perp LM$
3.3	$m_{KN} = \frac{10-2}{4-8}$ $= -2$ $\therefore KN \perp NM$ $\therefore \hat{KLM} + \hat{KNM} = 180^\circ$ $\therefore KLMN$ is cyclic quadrilateral (converse, opp $\angle^s$ of a cyclic quad are supplementary)

QUESTION 4

4.1	$M\left(\frac{-5+3}{2}; \frac{4+2}{2}\right) = M(-1; 3)$
4.2	$r^2 = BM^2 = (-5+1)^2 + (4-3)^2 = 17$ $\therefore (x+1)^2 + (y-3)^2 = 17$
4.3	$m_{AB} = \frac{2-3}{3+1} = -\frac{1}{4}$ $m_{AN} = \frac{2+2}{3-2} = 4$ $m_{AB} \times m_{AN} = -1$ $\therefore \hat{BAT} = 90^\circ$ $\therefore TA$ is a tangent (conv. tangent and diameter)
4.4.1	$m_{TA} = m_{AN} = 4$ $y = 4x + c$ Subst. (3; 2): $2 = 4(3) + c$ $-10 = c$ $\therefore y = 4x - 10$
4.4.2	Let C(x; y) $\therefore (x+1)^2 + (y-3)^2 = 17$ At C; $x = 0$ $\therefore (0+1)^2 + (y-3)^2 = 17$ $(y-3)^2 = 16$ $y-3 = \pm 4$ $y = 7$ or $y = -1$ $\therefore C(0; -1)$ $m_{BC} = \frac{-1-4}{0+5} = -1$ Now $y = -x - 1$
4.5	Lines AT and BT intersect at C $\therefore 4x - 10 = -x - 1$ $5x = 9$ $x = \frac{9}{5} = a$ $b = -\frac{9}{5} - 1 = -2\frac{4}{5}$

PAPER B

3.1

3.1.1	$m_{KL} = \frac{6 - 4}{3 - (-1)}$ $= \frac{2}{4} \text{ or } \frac{1}{2}$
3.1.2	midpoint of R : $\left(\frac{x_N + x_L}{2} ; \frac{Y_N + Y_L}{2} \right)$: $\left(\frac{3 + 4}{2} ; \frac{6 + 1}{2} \right)$ R $\left(\frac{7}{2} ; \frac{7}{2} \right)$
3.1.3	co-ordinates of M: $\frac{x - 1}{2} = \frac{7}{2}$ $x - 1 = 7 \quad \therefore x = 8$ $\frac{y + 4}{2} = \frac{7}{2}$ $y = 7 - 4 \quad \therefore y = 3$ $\therefore M(8; 3)$
3.2	The equation of NM $m_{NM} = \frac{1}{2} \quad [NM \parallel KL]$ $y - y_1 = m(x - x_1)$ $y - 1 = \frac{1}{2}(x - 4)$ $y = \frac{1}{2}x - 2 + 1 \therefore y = \frac{1}{2}x - 1$

3.3

3.3.1	$0 = \frac{1}{2}x - 1$ $x = 2$ $\therefore P(2; 0)$	
3.3.2	$\tan \theta = \frac{1}{2}$ $\theta = 26,57^\circ$	
3.3.3	$m_{KP} = \frac{0 - 4}{2 - (-1)} = -\frac{4}{3}$ $\tan^{-1}\left(\frac{-4}{3}\right) = 180^\circ - 53,13^\circ = 126,87^\circ$ $\hat{KPN} = 126,87^\circ - 26,57^\circ$ $= 100,3^\circ$	

Question 4

4.1.	$x^2 + 2x + y^2 - 4y = 27$ $x^2 + 2x + 1 + y^2 - 4y + 4 = 27 + 1 + 4$ $(x + 1)^2 + (y - 2)^2 = 32$ $\therefore T(-1; 2)$ OR $x = \frac{2}{-2} = -1$ $y = \frac{-4}{-2} = 2$
4.2	$m_{TB} \times m_{BD} = -1 \quad [\text{tangent} \perp \text{radius}]$ $m_{TB} = 1$ $y = mx + c$ $2 = 1 \cdot (-1) + c$ $\therefore c = 3$ $\therefore \text{Equation of TB is } y = x + 3$
4.3	equating TB and BD: $x + 3 = -x + 5$ $2x = 2$ $x = 1$ $y = 1 + 3 \quad \text{or} \quad y = -1 + 5$ $\therefore B(1; 4)$

4.4	$(x+1)^2 + (y-2)^2 = r^2$ $(1+1)^2 + (4-2)^2 = r^2$ $8 = r^2$ $(x+1)^2 + (y-2)^2 = 8$
4.5	Draw a horizontal line TE with E on BD: Point D(5 ; 0) and Point E: $2 = -x + 5$ $\therefore E(3 ; 2)$ Area of trap TEDO = $\frac{1}{2}(\text{TE} + \text{OD}) \times \perp h$ $= \frac{1}{2}(4 + 5) \times 2$ $= 9 \text{ units}$ Area of $\Delta \text{TBE} = \frac{1}{2}(\text{TB} \times \text{BE})$ $= \frac{1}{2}(\sqrt{8})(\sqrt{8})$ $= 4 \text{ units}$ Area of OTBD = $9 + 4$ $= 13 \text{ units}^2$

PAPER C

3.1	$E\left(\frac{12}{2}; \frac{6}{2}\right)$ $E(6; 3)$
3.2	$m_{BA} = \frac{6-0}{7-5}$ $= 3$ $y = mx + c$ $y = 3x + c$ $6 = 3(7) + c \quad \text{OR / OF}$ $c = -15$ $y = 3x - 15$ $y - y_1 = m(x - x_1)$ $y - 6 = 3(x - 7)$ $y = 3x - 21 + 6$ $y = 3x - 15$

3.3	$rx - 3y + 5 = 0$ $-3y = -rx - 5$ $y = \frac{r}{3}x + \frac{5}{3}$ $3 = \frac{r}{3}$ $r = 9$
3.4	Area $\Delta AOP = 10$ $\frac{1}{2} \times AO \times \perp h = 10$ $\frac{1}{2} \times 5 \times \perp h = 10$ $\perp h = 4$ but / maar $y < 0$ $\therefore y = -4$ $AP = BP$ $AP^2 = BP^2$ $(x-5)^2 + (-4-0)^2 = (x-7)^2 + (-4-6)^2$ $x^2 - 10x + 25 + 16 = x^2 - 14x + 49 + 100$ $4x = 108$ $x = 27$ P (27 ; -4)

Question 4

4.1	$\alpha = 135^\circ$ ext $\angle$ of Δ / buite $\angle$ van Δ $\tan(135^\circ) = m$ $m = -1$ $y = mx + c$ $y = -1x + c$ $4 = -1(-2) + c$ OR / OF $y - y_1 = m(x - x_1)$ $c = 2$ $y - 4 = -1(x + 2)$ $y = -x + 2$ $y = -1x - 2 + 4$ $y = -x + 2$
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4.2	$P(-4; 0)$ $a = -4$ $m_{BA} \cdot m_{AS} = -1$ $m_{BA} = 1$ $m_{BA} = \frac{4-b}{-2+4}$ $1 = \frac{4-b}{2}$ $2 = 4-b$ $b = 2$
4.3	$(x-a)^2 + (y-b)^2 = r^2$ $(x+4)^2 + (y-2)^2 = r^2$ $(-2+4)^2 + (4-2)^2 = r^2$ $4+4 = r^2$ $(x+4)^2 + (y-2)^2 = 8$
4.4	$x^2 - 2x + y^2 - 2y = 0$ $(x^2 - 2x + 1) + (y^2 - 2y + 1) = 1 + 1$ $(x-1)^2 + (y-1)^2 = 2$
4.5	D (1 ; 1)
4.6	$DE = \sqrt{2}$ $DA = \sqrt{(-2-1)^2 + (4-1)^2}$ $= \sqrt{9+9}$ $= \sqrt{18} \quad \text{OR/OR} \quad = 3\sqrt{2}$ $\hat{DEA} = 90^\circ$ radius $\perp$ tangent $AD^2 = DE^2 + AE^2$ pythagoras $(\sqrt{18})^2 = (\sqrt{2})^2 + AE^2$ $18 - 2 = AE^2$ $AE = 4$

PAPER D

3.1.1	$1 = \frac{3+x}{2} \qquad -2 = \frac{4+y}{2}$ $2 = 3+x \qquad -4 = 4+y$ $x = -1 \qquad y = -8$ $B(-1; -8)$
3.1.2	$m_{CD} = \frac{0-4}{6-3}$ $= -\frac{4}{3}$
3.1.3	$y-2 = \frac{-4}{3}(x-11)$ $y = \frac{-4}{3}x + \frac{50}{3}$ OR / OF $y = \frac{-4}{3}x + c$ $2 = \frac{-4}{3}(11) + c$ $c = \frac{50}{3}$ $y = \frac{-4}{3}x + \frac{50}{3}$

3.1.4

$$\begin{aligned} CD &= \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} \quad \text{OR / OF} \\ &= \sqrt{(0 - 4)^2 + (6 - 3)^2} \quad R(5;10) \text{ midpoint / middelpunt} \\ &= \sqrt{25} \quad RQ = \sqrt{(2 - 10)^2 + (11 - 5)^2} \\ CD &= 5 \quad RQ = 10 \end{aligned}$$

D is the midpoint of PR / *D is die middelpunt van PR*

C is the midpoint of PQ (line from midpoint of 1 side || to 2nd side) /

C is die middelpunt van PQ (lyn van middelpunt van 1 sy || aan 2de sy)

$$RQ = 2CD = 10 \quad (\text{midpoint theorem / middelpuntstelling})$$

$$PK = RQ$$

$$\sqrt{(y+2)^2 + (4-1)^2} = 10$$

$$\sqrt{(y+2)^2 + (4-1)^2} = 10$$

$$(y+2)^2 + (4-1)^2 = 10^2$$

$$y^2 + 4y + 4 + 9 = 100$$

$$(y+2)^2 = 91 \quad \text{or / of}$$

$$y^2 + 4y - 87 = 0$$

$$y+2 = \pm\sqrt{91}$$

$$y = \frac{-4 \pm \sqrt{4^2 - 4(1)(-87)}}{2(1)}$$

$$y = \pm\sqrt{91} - 2$$

$$y = \frac{-4 \pm \sqrt{364}}{2}$$

$$y = -11,54$$

or / of

$$y \neq 7,54$$

$$y = -11,54 \quad \text{or / of} \quad y \neq 7,54$$

3.2.1

$$m_{PQ} = \tan \theta$$

$$\tan \theta = 1$$

$$\theta = 45^\circ$$

$$\hat{P}_1 = 35^\circ$$

vertical opp $\angle$ s / regoorst $\angle$ e

QR || to the x-axis / aan die x-as

$$\hat{T}_1 = 35^\circ + 45^\circ$$

ext $\angle$ of Δ / buite $\angle$ v Δ

$$\hat{T}_1 = 80^\circ$$

$$\alpha = \hat{T}_1 = 80^\circ$$

corr $\angle$ s ST || QR /
ooreenkomstige $\angle$ e ST || QR

3.2.2	$\frac{U_2 + (-8)}{2} = 1$ x at/by U: $\therefore U_x = 10 \text{ units / eenhede}$ $QU = 18 \text{ units / eenhede}$ x at/by W = x at / by U = 10 y at/by W: $y = 10 + \frac{2}{3}$ $= \frac{32}{3}$ $WU = \frac{32}{3} + 5 = \frac{47}{3}$ $\therefore \text{Area } \Delta QWU = \frac{1}{2}(18)\left(\frac{47}{3}\right)$ $= 141 \text{ square units / eenhede kwadraat}$
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Question 4

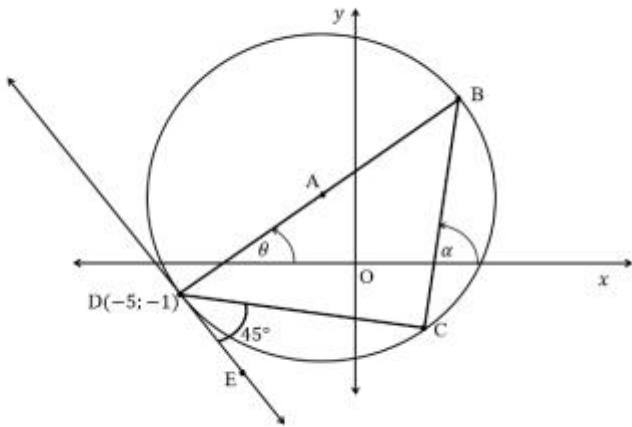
4.1	$\hat{TUS} = 180^\circ - 101,31^\circ = 78,69^\circ \quad \text{adj supp } \angle s /$ <i>aangrensende suppl \angle e</i> $m_{TU} = \tan 78,69^\circ = 5$ $c = 6$ $y = 5x + 6$
4.2	$x - \text{int / afsnit } y = 0$ $\frac{-1}{5}x + \frac{4}{5} = 0$ $-x + 4 = 0$ $x = 4$ $\therefore S(4;0)$ $M = \left(\frac{-6+4}{2}; \frac{2+0}{2} \right)$ $\therefore M(-1;1)$

4.3	$(x+1)^2 + (y-1)^2 = r^2$ $(-6+1)^2 + (2-1)^2 = r^2$ $r^2 = 26$ $(x+1)^2 + (y-1)^2 = 26$ OR / OF $(x+1)^2 + (y-1)^2 = r^2$ $(4+1)^2 + (0-1)^2 = r^2$ $r^2 = 26$ $(x+1)^2 + (y-1)^2 = 26$
4.4	$m_{MP} = -\frac{1}{5} \qquad m_{MP} \times m_{KL} = -1$ $m_{KL} = 5 \qquad \text{radius} \perp \text{tan} / \text{radius} \perp \text{raaklyn}$ $m_{TU} = 5 \qquad \text{proven / reeds bewys}$ $\therefore m_{TU} = m_{KL} = 5$ $KL \parallel TU$
4.5	$VM = \sqrt{\left(-1 + \frac{1}{2}\right)^2 + (1-7)^2}$ $= 6,02$ $\text{radius} = \sqrt{26} = 5,1$ $6,02 > 5,1$ $\therefore V\left(-\frac{1}{2}; 7\right)$ does not lie within the circle / <i>lê nie binne die sirkel nie.</i>

PAPER E

3.1	$AC=BD$ $\sqrt{50} = \sqrt{(3 - (-2))^2 + (p - (-4))^2}$ $50 = 25 + p^2 + 8p + 16$ $p^2 + 8p - 9 = 0$ $(p + 9)(p - 1) = 0$ $p \neq -9; p = 1$
3.2	$M\left(\frac{3-2}{2}; \frac{1-4}{2}\right)$ $M\left(\frac{1}{2}; -\frac{3}{2}\right)$
3.3	$m_{DC} = \frac{1 - (-2)}{3 - 4}$ $m_{DC} = \frac{3}{-1}$ $m_{DC} = -3$
3.4	$y = -3x + c$ $-4 = -3(-2) + c$ $-10 = c$ $y = -3x - 10$

QUESTION 4

		
4.1.1	$A(-1; 2)$	

4.1.2	$\frac{x + (-5)}{2} = -1$ $x - 5 = -2$ $x = 3$ $\frac{y + (-1)}{2} = 2$ $y - 1 = 4$ $y = 5$ $B(3; 5)$
4.1.3	$m_{AD} = \frac{2 - (-1)}{-1 - (-5)}$ $= \frac{3}{4}$
4.1.4	$\tan \theta = \frac{3}{4}$ $\theta = 36,87^\circ$
4.1.5	$m_{radius} = \frac{3}{4}$ $m_{tangent} = \frac{-4}{3} \quad \text{radius} \perp \text{tangent}$ $y = -\frac{4}{3}x + c$ $-1 = -\frac{4}{3}(-5) + c$ $-1 = \frac{20}{3} + c$ $c = -\frac{23}{3}$ $\therefore y = -\frac{4}{3}x - \frac{23}{3}$

4.2	$\hat{B} = 45^\circ$ [tan; chord theorem / raaklyn; koord stelling] $\alpha = 45^\circ + 36,87^\circ$ [ext $\angle$ of Δ / buite $\angle$ van Δ] $\alpha = 81,87^\circ$ $m_{BC} = \tan 81,87^\circ$ $m_{BC} = 7$
4.3.1	$x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$ $(x - 3)^2 + (y + 1)^2 = 18$ $\therefore M(3; -1)$
4.3.2	$M(3; -1)$ and $A(-1; 2)$ $MA = \sqrt{(3 - (-1))^2 + (-1 - 2)^2}$ $MA = \sqrt{16 + 9}$ $MA = \sqrt{25}$ $MA = 5$ $r_M + r_A = \sqrt{18} + 5$ or $= 9,24$ $MA < r_M + r_A$ $\therefore$ circles intersect / sirkels sny

PAPER F

2.2.1	$x^2 + 2x + \left(\frac{1}{2}(2)\right)^2 + y^2 - 4y + \left(\frac{1}{2}(-4)\right)^2 = 5 + 1 + 4$ $(x + 1)^2 + (y - 2)^2 = 10$ But $(x; y) \rightarrow (x - 2; y + 4)$ $\therefore (-1; 2) \rightarrow (-1 - 2; 2 + 4)$ $= (-3; 6)$ $\therefore (x + 3)^2 + (y - 6)^2 = 10$ OR $(x + 2)^2 + 2(x + 2) + (y - 4)^2 - 4(y - 4) - 5 = 0$ $x^2 + y^2 + 6x - 12y + 35 = 0$ $(x + 3)^2 + (y - 6)^2 = 10$
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2.2.2	Distance from origin to centre $= \sqrt{(-3-0)^2 + (6-0)^2} = \sqrt{45}$ Since $\sqrt{45} > \sqrt{10}$, the origin lies outside the circle.
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Question 3

3.1	$AB : y + 3x - 2 = 0$ $\therefore y = -3x + 2$ $\therefore k = 2$
3.2	$AC = \sqrt{(1-6)^2 + (-1-4)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$
3.3	$m_{AB} = \frac{-1-2}{1-0} = -3$ $m_{BC} = \frac{4-2}{6-0} = \frac{1}{3}$ $m_{AB} \times m_{BC} = -3 \times \frac{1}{3} = -1$ $\therefore \hat{ABC} = 90^\circ$.
3.4	$\tan \beta = m_{AC} = 1$ $\beta = 45^\circ$ $\tan \phi = m_{BC} = \frac{1}{3}$ $\phi = 18,43^\circ$ $\theta = 45^\circ - 18,43^\circ$ [ext $\angle$ of Δ] $= 26,57^\circ$

QUESTION 5

5.1.1

$$x^2 + y^2 - 8x + 6y$$

$$= (2)^2 + (-9)^2 - 8(2) + 6(-9)$$

$$= 4 + 81 - 16 - 54$$

$$= 15$$

Hence, the point lies on the circumference of the circle

OR

$$x^2 + y^2 - 8x + 6y = 15$$

$$(x - 4)^2 + (y + 3)^2 = 15 + 16 + 9$$

$$(x - 4)^2 + (y + 3)^2 = 40$$

$$(x - 4)^2 + (y + 3)^2$$

$$= (2 - 4)^2 + (-9 + 3)^2$$

$$= 2^2 + 6^2$$

$$= 40$$

∴ The point lies on the circumference of the circle.

5.1.2

$$x^2 + y^2 - 8x + 6y = 15$$

$$(x - 4)^2 + (y + 3)^2 = 15 + 16 + 9$$

$$(x - 4)^2 + (y + 3)^2 = 40$$

Circle centre (4 ; -3)

$$m_{rad} = \frac{-3 - (-9)}{4 - 2}$$

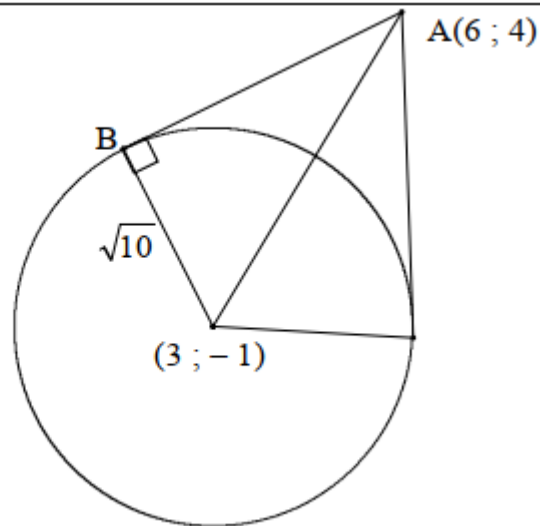
$$m_{rad} = 3$$

$$m_{tan} = -\frac{1}{3}$$

$$y + 9 = -\frac{1}{3}(x - 2)$$

$$y = -\frac{1}{3}x - \frac{25}{3}$$

5.2



Radius $r = \sqrt{10}$

Distance from A to centre of circle is

$$= \sqrt{(6-3)^2 + (4+1)^2}$$

$$= \sqrt{9+25}$$

$$= \sqrt{34}$$

$$AB^2 = 34 - 10$$

$$AB^2 = 24$$

$$AB = \sqrt{24}$$

$$AB = 2\sqrt{6}$$

$$AB = 4,90$$

OR

$$r^2 = 10$$

$$r = \sqrt{10}$$

Radius $\perp$ tangent

By Pythagoras

$$AB^2 = (6-3)^2 + (4+1)^2 - 10$$

$$= 24$$

$$AB = 4,90$$

QUESTION 6

6.1	$x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + 8x + 16 + y^2 + 4y + 4 = 16 + 4 + 38$ $(x + 4)^2 + (y + 2)^2 = 58$ Centre is $(-4 ; -2)$ and the radius is $\sqrt{58}$
6.2	Centre of second circle is $(4 ; 6)$ Distance between centres is $\sqrt{(4 + 4)^2 + (6 + 2)^2} = \sqrt{128} = 11,31$
6.3	Sum of radii $= \sqrt{58} + \sqrt{26} = 12,71$ Distance between centres is 11,31. sum of the radii $>$ distance between the centres $\therefore$ the circles must overlap and hence the circles must intersect.
6.4	Equation of second circle: $(x - 4)^2 + (y - 6)^2 = 26$ $x^2 - 8x + 16 + y^2 - 12y + 36 = 26$ $x^2 - 8x + y^2 - 12y + 26 = 0$ Let $(x ; y)$ be either of the two points on intersection. Then $x^2 + y^2 + 8x + 4y - 38 = 0$ and $x^2 + y^2 - 8x - 12y + 26 = 0$ Subtract $\frac{16y + 16x - 64 = 0}{y = -x + 4}$ Both points of intersection lie on this line. $\therefore y = -x + 4$ is the equation of the common chord. OR

Check that the line $y = -x + 4$ cuts the two circles at the same points:

$$(x-4)^2 + (-x-2)^2 = 26$$

$$x^2 - 8x + 16 + x^2 + 4x + 4 = 26$$

$$2x^2 - 4x - 6 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \text{ or } x = -1$$

$$x^2 + y^2 + 8x + 4y - 38 = 0$$

$$x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$$

$$x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$$

$$2x^2 - 4x - 6 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x = 3 \text{ or } x = -1$$

TRIGONOMETRY**PAPER A****QUESTION 5**

5.1	$\begin{aligned} &\cos 79^\circ \cos 311^\circ + \sin 101^\circ \sin 49^\circ \\ &= \cos 79^\circ \cos 49^\circ + \sin 79^\circ \sin 49^\circ \\ &= \cos(79^\circ - 49^\circ) \\ &= \cos 30^\circ \\ &= \frac{\sqrt{3}}{2} \end{aligned}$ Answer only no marks, used calculator
5.2	$\sin(x + y) = 3 \sin(x - y)$ $\begin{aligned} \sin x \cos y + \cos x \sin y &= 3(\sin x \cos y - \cos x \sin y) \\ \sin x \cos y + \cos x \sin y &= 3 \sin x \cos y - 3 \cos x \sin y \\ -2 \sin x \cos y &= -4 \cos x \sin y \end{aligned}$ ÷ $-2 \cos x \cos y$: $\frac{\sin x}{\cos x} = 2 \left(\frac{\sin y}{\cos y} \right)$ $\therefore \tan x = 2 \tan y$
5.3.1	$\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2 \sin x} = \sin x$ LHS: $\begin{aligned} &\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2 \sin x} \\ &= \frac{\cos x}{2 \sin x \cos x} - \frac{1 - 2 \sin^2 x}{2 \sin x} \\ &= \frac{1}{2 \sin x} - \frac{(1 - 2 \sin^2 x)}{2 \sin x} \\ &= \frac{2 \sin x}{2 \sin x} \\ &= \sin x \\ &= \text{RHS} \end{aligned}$

5.3.2	$1 + 2 \cos 2x = \frac{\cos 2x}{2 \sin x} - \frac{\cos x}{\sin 2x}$ $1 + 2 \cos 2x = -\sin x$ $1 + 2(1 - 2\sin^2 x) = -\sin x$ $1 + 2 - 4\sin^2 x = -\sin x$ $4\sin^2 x - \sin x - 3 = 0$ $(\sin x - 1)(4 \sin x + 3) = 0$ $\sin x = 1 \quad \text{OR} \quad \sin x = -\frac{3}{4}$ $x = 90^\circ \quad \text{ref} \angle = 48,59^\circ$ $x = 228,59^\circ$ OR $x = 311,41^\circ$
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QUESTION 6

6.1	$a = 1$ $b = 2$ $c = 2$ $d = 1$
6.2	360°
6.3.1	$x \in [-90^\circ; 90^\circ] \quad \text{or} \quad x \in [270^\circ; 360^\circ]$
6.3.2	$x \in (-45^\circ; 0^\circ) \quad \text{or} \quad x \in (45^\circ; 90^\circ) \quad \text{or} \quad x \in (315^\circ; 360^\circ)$

QUESTION 7

7.1	n ΔPQR: $\hat{Q}_1 = x \quad (PR = QR)$ $\hat{R} = 180^\circ - 2x \quad (\text{sum of } \angle \Delta PQR)$ $\text{Area of } \Delta PQR = \frac{1}{2}pq \sin \hat{R}$ $= \frac{1}{2}m \cdot m \sin(180^\circ - 2x)$ $= \frac{1}{2}m^2 \sin 2x$
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7.2	$\therefore \frac{PQ}{\sin(180^\circ - 2x)} = \frac{m}{\sin x}$ $\therefore PQ = \frac{m \cdot \sin(180^\circ - 2x)}{\sin x}$ $\therefore PQ = \frac{m \cdot \sin 2x}{\sin x}$ $\therefore PQ = \frac{m \cdot 2 \sin x \cdot \cos x}{\sin x}$ $\therefore PQ = 2m \cos x$
7.3	In $\triangle SPQ$: $\tan y = \frac{SP}{PQ}$ $\therefore SP = PQ \tan y$ $\therefore SP = 2m \cos x \tan y$

PAPER B

QUESTION 5

5.1.1	$\tan \theta = \frac{-\sqrt{3}}{1} = -\sqrt{3}$
5.1.2	$r^2 = (1)^2 + (-\sqrt{3})^2 \quad \text{Pyth}$ $r^2 = 4$ $r = 2$ $\sin(-\theta) = -\sin \theta$ $= -\left(\frac{-\sqrt{3}}{2}\right) \text{ or } \frac{\sqrt{3}}{2}$
5.1.3	$\sin(\theta - 60^\circ)$ $= \sin \theta \cos 60^\circ - \cos \theta \sin 60^\circ$ $= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{-\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$

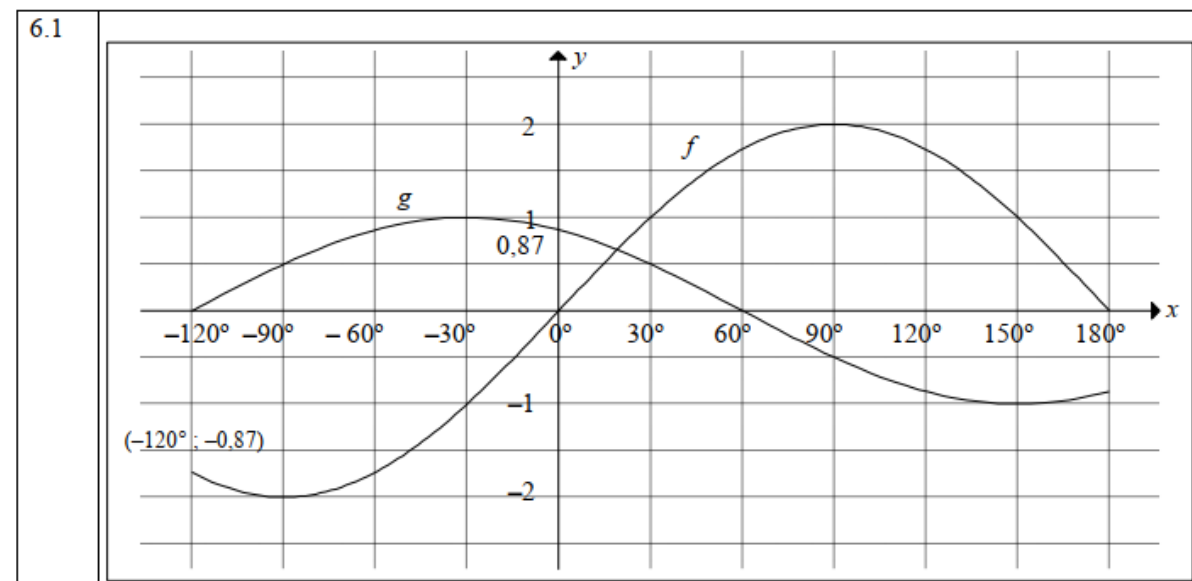
$$\begin{aligned}
 5.2 \quad & \frac{\tan(180^\circ - \theta) \sin(90^\circ + \theta)}{\cos 300^\circ \sin(\theta - 360^\circ)} \\
 &= \frac{-\tan \theta \cdot \cos \theta}{\cos 60^\circ \cdot \sin \theta} \\
 &= \frac{-\frac{\sin \theta}{\cos \theta} \cdot \cos \theta}{\frac{1}{2} \cdot \sin \theta} \\
 &= -2
 \end{aligned}$$

$$\begin{aligned}
 5.3.1 \quad & LHS = \frac{\cos 2x - 1}{\sin 2x} \\
 &= \frac{1 - 2\sin^2 x - 1}{2\sin x \cos x} \\
 &= -\frac{\sin x}{\cos x} \\
 &= -\tan x \\
 &= RHS
 \end{aligned}$$

$$5.3.2 \quad x = 90^\circ ; 180^\circ ; 270^\circ$$

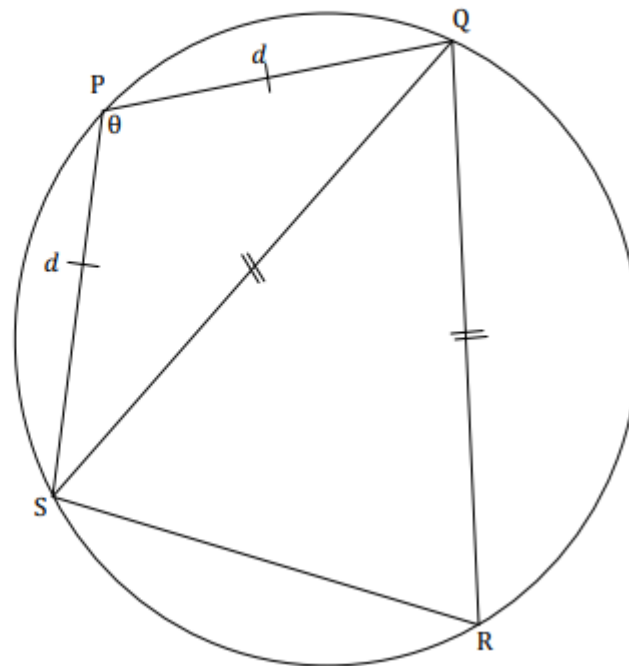
$$\begin{aligned}
 5.3.3 \quad & -\tan 2x = \frac{1}{4} \\
 & \tan 2x = -\frac{1}{4} \\
 & 2x = 165,96^\circ + k \cdot 180^\circ \\
 & x = 82,98^\circ + k \cdot 90^\circ ; k \in \mathbb{Z}
 \end{aligned}$$

QUESTION 6



6.2	Period = 360°
6.3.1	$-30^\circ < x < 150^\circ$
6.3.2	$x \in (0^\circ ; 60^\circ)$
6.4	$y = \cos(x + 30^\circ + 60^\circ)$ $y = \cos(x + 90^\circ)$ $y = -\sin x$

QUESTION 7



7.1	$QS^2 = d^2 + d^2 - 2d \cdot d \cdot \cos \theta$ $QS^2 = 2d^2 - 2d^2 \cdot \cos \theta$ $QS^2 = 2d^2(1 - \cos \theta)$ $QS = d\sqrt{2(1 - \cos \theta)}$
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7.2	$\hat{R} = 180^\circ - \theta$ opp. $\angle^s$ cyclic quad suppl $= \hat{Q}\hat{S}R$ equal sides, equal angles $S\hat{Q}R = 2\theta - 180^\circ$ sum $\angle^s \Delta$ $\Delta QRS = \frac{1}{2} \cdot QS \cdot QR \sin S\hat{Q}R$ $= \frac{1}{2} \cdot d\sqrt{2(1 - \cos \theta)} \cdot d\sqrt{2(1 - \cos \theta)} \sin(2\theta - 180^\circ)$ $= \frac{1}{2} d^2 \cdot 2(1 - \cos \theta)(-\sin 2\theta)$ $= -d^2(1 - \cos \theta) \cdot \sin 2\theta$
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PAPER C

QUESTION / VRAAG 5

5.1	$1 - 4 \sin^2 15^\circ$ $= 1 - 4 \sin^2 (45^\circ - 30^\circ)$ $= 1 - 4 [\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ]^2$ $= 1 - 4 \left[\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \right]^2$ $= 1 - 4 \left[\frac{\sqrt{6} - \sqrt{2}}{4} \right]^2$ $= 1 - 4 \left[\frac{6 - 4\sqrt{3} + 2}{16} \right]$ $= 1 - 4 \left[\frac{8 - 4\sqrt{3}}{16} \right]$ $= 1 - \left[\frac{8 - 4\sqrt{3}}{4} \right]$ $= \sqrt{3} - 1$
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$$\begin{aligned}
 5.2 \quad & \frac{\sqrt{3} \sin x \cdot \sin^2 72^\circ + \sin^2 198^\circ \cdot \sqrt{3} \cos(x - 90^\circ)}{\tan 120^\circ \cdot \sin x} \\
 &= \frac{\sqrt{3} \sin x \cdot \sin^2 (90^\circ - 18^\circ) + \sin^2 (180^\circ + 18^\circ) \cdot \sqrt{3} \sin x}{\tan (180^\circ - 60^\circ) \cdot \sin x} \\
 &= \frac{\sqrt{3} \sin x \cdot \cos^2 18^\circ + \sin^2 18^\circ \cdot \sqrt{3} \sin x}{-\tan 60^\circ \cdot \sin x} \\
 &= \frac{\sqrt{3} \sin x (\cos^2 18^\circ + \sin^2 18^\circ)}{-\sqrt{3} \cdot \sin x} \\
 &= -1
 \end{aligned}$$

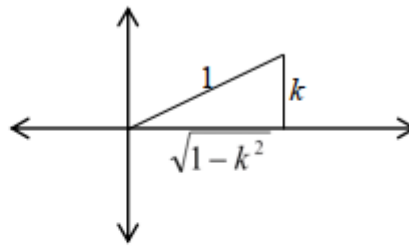
$$\begin{aligned}
 5.3 \quad & 6 \sin x \cdot \cos x + 3 \cos x - 4 \sin^2 x - 2 \sin x = 0 \\
 & 3 \cos x (2 \sin x + 1) - 2 \sin x (2 \sin x + 1) = 0 \\
 & (2 \sin x + 1)(3 \cos x - 2 \sin x) = 0 \\
 & \sin x = -\frac{1}{2} \quad \text{OR/OR} \quad 3 \cos x = 2 \sin x \\
 & \qquad \qquad \qquad \tan x = \frac{3}{2} \\
 & \text{RA} = 30^\circ \qquad \qquad \text{RA} = 56,31^\circ \\
 & x = 210^\circ + k \cdot 360^\circ \qquad x = 56,31^\circ + k \cdot 180^\circ \\
 & x = 330^\circ + k \cdot 360^\circ \qquad x = 236,31^\circ + k \cdot 180^\circ; k \in \mathbb{Z}
 \end{aligned}$$

$$\begin{aligned}
 5.4 \quad & (1 - \tan A) \left(\frac{\cos A}{\cos 2A} \right) = \frac{1}{\cos A + \sin A} \\
 & \text{LHS/LK} = (1 - \tan A) \left(\frac{\cos A}{\cos 2A} \right) \\
 &= \left(1 - \frac{\sin A}{\cos A} \right) \left(\frac{\cos A}{\cos^2 A - \sin^2 A} \right) \\
 &= \left(\frac{\cos A - \sin A}{\cos A} \right) \left(\frac{\cos A}{(\cos A - \sin A)(\cos A + \sin A)} \right) \\
 &= \frac{1}{\cos A + \sin A} \\
 & \text{LHS / LK} = \text{RHS / RK}
 \end{aligned}$$

5.5.1

$$\cos 2\theta$$

$$= \sqrt{1 - k^2}$$



OR/OF

$$\cos^2 2\theta = 1 - \sin^2 2\theta$$

$$= 1 - k^2$$

$$\cos 2\theta = \sqrt{1 - k^2}$$

5.5.2

$$\frac{\sin 2\theta}{\tan \theta}$$

$$\frac{\sin 2\theta}{\tan \theta}$$

$$= \frac{2 \sin \theta \cdot \cos \theta}{\frac{\sin \theta}{\cos \theta}}$$

$$= 2 \sin \theta \cdot \cos \theta \cdot \frac{\cos \theta}{\sin \theta}$$

$$= 2 \cos^2 \theta$$

But/maar $\cos 2\theta = \sqrt{1 - k^2}$

$$2 \cos^2 \theta - 1 = \sqrt{1 - k^2}$$

$$2 \cos^2 \theta = \sqrt{1 - k^2} + 1$$

$$\frac{\sin 2\theta}{\tan \theta} = \sqrt{1 - k^2} + 1$$

QUESTION / VRAAG 6

6.1	$a = -1$ $d = 2$
6.2	$D\left(-150^\circ; \frac{1}{2}\right)$
6.3.1	$-90^\circ < x < 90^\circ$ OR / OF $x \in (-90^\circ; 90^\circ)$
6.3.2	$-135^\circ < x < -45^\circ$ OR / OF $x \in (-135^\circ; -45^\circ)$

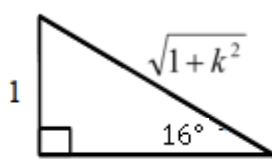
Question 7

7.1	In $\triangle KLM$ $\frac{KM}{LM} = \tan x$ $\frac{KM}{r} = \tan x$ $KM = r \tan x$ In $\triangle KMN$ $\frac{MN}{KM} = \tan x$ $\frac{2r}{KM} = \tan x$ $\frac{2r}{r \tan x} = \tan x$ $2 = \tan^2 x$ $\sqrt{2} = \tan x$ $x = 54,74^\circ$	
	OR/OF $KM = \frac{2r}{\tan x}$ $r \tan x = \frac{2r}{\tan x}$	

7.2	$LN^2 = LM^2 + MN^2 - 2 LM \cdot MN \cos M$ $LN^2 = (5)^2 + (10)^2 - 2(5) \cdot (10) \cos 110^\circ$ $LN^2 = 159,20$ $LN = \sqrt{159,20}$ $LN = 12,62 m$
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PAPER D

QUESTION / VRAAG 5

5.1.1	$x^2 = (\sqrt{1+k^2})^2 - (1)^2 \quad (\text{Pythagoras})$ $x^2 = k^2$ $x = k$ $\tan 16^\circ = \frac{1}{k}$	
5.1.2	$\cos 32^\circ$ $= \cos 2(16^\circ)$ $= 2 \cos^2 16^\circ - 1$ $= 2 \left(\frac{k}{\sqrt{1+k^2}} \right)^2 - 1$	
5.2	$\frac{\cos(90^\circ + x) \sin(x - 180^\circ) - \cos^2(180^\circ - x)}{\cos(-2x)}$ $= \frac{(-\sin x)(-\sin x) - \cos^2 x}{\cos 2x}$ $= \frac{\sin^2 x - \cos^2 x}{\cos^2 x - \sin^2 x} \quad \text{OR / OF} \quad \frac{-\cos 2x}{\cos 2x}$ $= \frac{-(\cos^2 x - \sin^2 x)}{\cos^2 x - \sin^2 x} \quad = -1$ $= -1$	

5.3	$\begin{aligned} & \cos 75^\circ \cdot \cos 45^\circ - \cos 15^\circ \cdot \cos 45^\circ \\ &= \cos 75^\circ \cdot \cos 45^\circ - \sin 75^\circ \cdot \sin 45^\circ \\ &= \cos(75^\circ + 45^\circ) \\ &= \cos 120^\circ \\ &= -\cos 60^\circ \\ &= -\frac{1}{2} \end{aligned}$
5.4.1	$\begin{aligned} & \tan \theta \left(\sin 2\theta + \frac{3 \cos^2 \theta}{\sin \theta} \right) \\ &= \frac{\sin \theta}{\cos \theta} \left(2 \sin \theta \cos \theta + \frac{3 \cos^2 \theta}{\sin \theta} \right) \\ &= 2 \sin^2 \theta + 3 \cos \theta \\ &= 2(1 - \cos^2 \theta) + 3 \cos \theta \\ &= -2 \cos^2 \theta + 3 \cos \theta + 2 \end{aligned}$
5.4.2	$\begin{aligned} -2 \cos^2 \theta + 3 \cos \theta + 2 &= 0 \\ 2 \cos^2 \theta - 3 \cos \theta - 2 &= 0 \\ (2 \cos \theta + 1)(\cos \theta - 2) &= 0 \\ \cos \theta &= -\frac{1}{2} \quad \text{or / of} \quad \cos \theta = 2 \\ &\text{no solution / geen oplossing} \\ &\text{ref / verw} \quad \angle = 60^\circ \\ \theta &= \pm 120^\circ + k360^\circ; k \in \mathbb{Z} \quad \text{OR / OF} \quad \theta = 120^\circ + k360^\circ; k \in \mathbb{Z} \\ &\quad \theta = 240^\circ + k360^\circ; k \in \mathbb{Z} \end{aligned}$
5.5	$\begin{aligned} \cos(a+b) &= -\frac{\sqrt{2}}{2} && \text{ref } \angle / \text{verw } \angle = 45^\circ \\ a+b &= 180^\circ - 45^\circ \\ a+b &= 135^\circ \dots\dots\dots (1) \\ \cos(a-2b) &= \frac{1}{2} && \text{ref } \angle / \text{verw } \angle = 60^\circ \\ a-2b &= 60^\circ \dots\dots\dots (2) \\ 3b &= 75^\circ && (1)-(2) \\ b &= 25^\circ \\ a &= 110^\circ \end{aligned}$

QUESTION 6

6.1	$x = -45^\circ$ $x = 135^\circ$
6.2	$h(x) = \tan(45^\circ - x)$ $h(x) = -\tan(x - 45^\circ) = -f(x)$ h is the reflection of f about the x -axis OR h is the reflection of f about the line $y = 0$
6.3	$y = 3 \sin 2x$

QUESTION 7

7.1	$\hat{MAG} = 2\theta$
7.2	$\frac{MG}{\sin 2\theta} = \frac{k}{\sin 90^\circ}$ $MG = \frac{k \sin 2\theta}{1} \dots \dots (1)$

7.3

$$\frac{MC}{\sin 150^\circ} = \frac{MG}{\sin \theta}$$

$$\begin{aligned} \frac{MC}{\frac{1}{2}} &= \frac{MG}{\sin \theta} \\ MC &= \frac{\frac{1}{2}MG}{\sin \theta} \dots \dots (1) \end{aligned}$$

Subst. (2) in (1)

$$MC = \frac{k \sin 2\theta}{2 \sin \theta}$$

$$MC = \frac{k 2 \sin \theta \cdot \cos \theta}{2 \sin \theta}$$

$$MC = k \cos \theta$$

7.4

$$\begin{aligned} \Delta MGC &= \frac{1}{2} MG \cdot CG \sin 150^\circ \\ \Delta MGC &= \frac{1}{2} k \sin 2\theta \cdot 8 \cdot \frac{1}{2} \\ \Delta MGC &= 2k \sin 2\theta \end{aligned}$$

OR

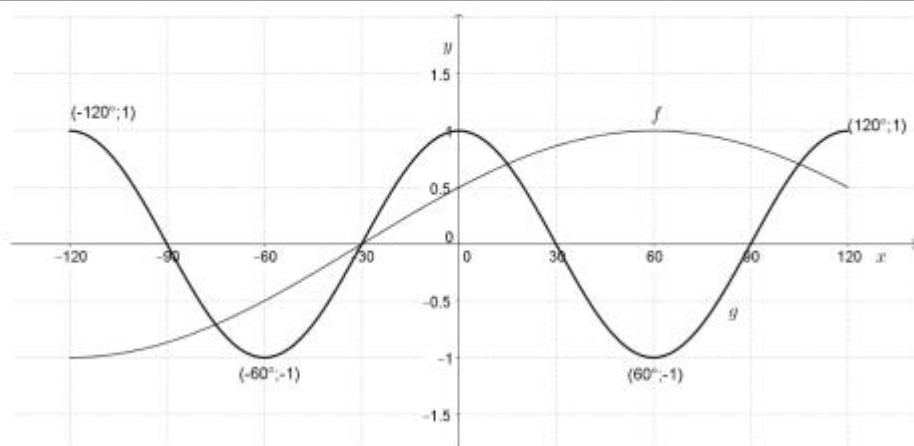
$$\begin{aligned} \Delta MGC &= \frac{1}{2} MC \cdot CG \sin \theta \\ \Delta MGC &= \frac{1}{2} k \cos \theta \cdot 8 \sin \theta \\ \Delta MGC &= 4k \cos \theta \sin \theta \\ \Delta MGC &= 2k(2 \cos \theta \sin \theta) \\ \Delta MGC &= 2k \sin 2\theta \end{aligned}$$

PAPER E

QUESTION 6

6.1

$$f(x) = \sin(x + a)$$

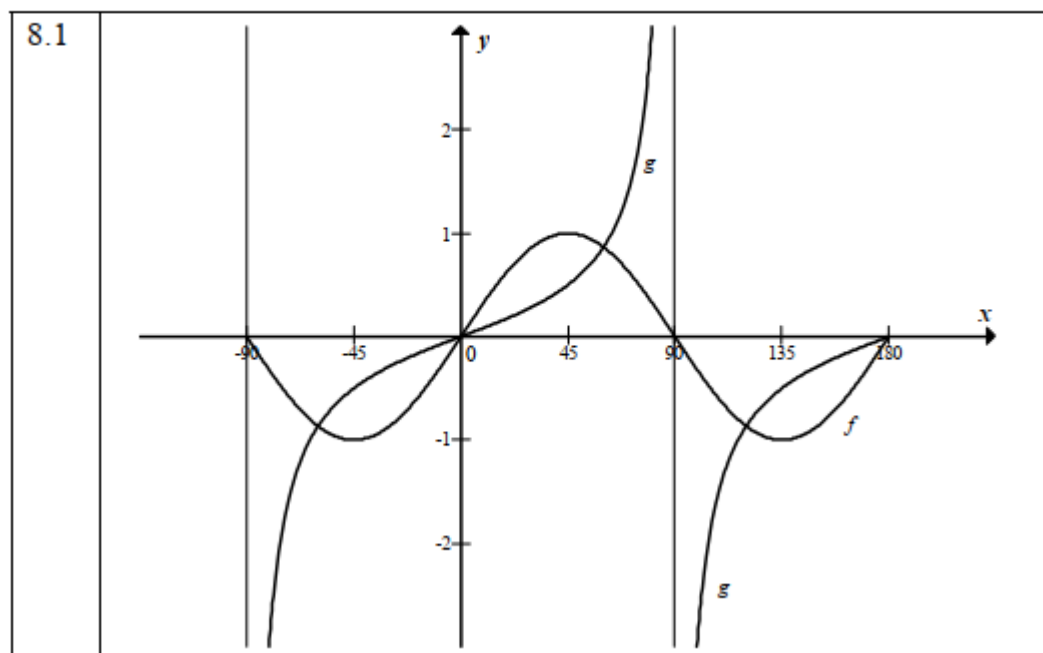


6.1	$a = 30^\circ$
6.2	Sketch
6.3	$f(x) = g(x)$ $\sin(x + 30^\circ) = \cos(3x)$ $\sin(x + 30^\circ) = \sin(90^\circ - 3x)$ $x + 30^\circ = 90^\circ - 3x + 360^\circ n \quad n \in \mathbb{Z}$ $4x = 60^\circ + 360^\circ n$ $x = 15^\circ + 90^\circ n$ <i>or</i> $x + 30^\circ = 180^\circ - (90^\circ - 3x) + 360^\circ n$ $x + 30^\circ = 90^\circ + 3x + 360^\circ n$ $-2x = 60^\circ + 360^\circ n$ $x = -30^\circ - 180^\circ n$
6.4	$15^\circ < x < 105^\circ$ or $x \in (15^\circ; 105^\circ)$
6.5	$g(x) = \cos(3x)$ $k(x) = \cos(60^\circ - 3x)$ $k(x) = \cos(3x - 60^\circ)$ $k(x) = \cos 3(x - 20^\circ)$ $\therefore$ translated 20° to the right.

QUESTION 7

7.1	$\frac{7}{PB} = \sin 18^\circ$ $PB = \frac{7}{\sin 18^\circ}$ $PB = 22,65 \text{ m} \quad (22,65247584\dots)$
7.2	$\frac{18}{PA} = \cos 23^\circ$ $PA = \frac{18}{\cos 23^\circ}$ $PA = 19,55 \text{ m} \quad (19,55448679\dots)$
7.3	$AB^2 = (22,65)^2 + (19,55)^2 - 2(22,65)(19,55) \cdot \cos 42^\circ$ $= 237,0847954\dots$ $AB = 15,40 \text{ m} \quad (15,3975581\dots)$

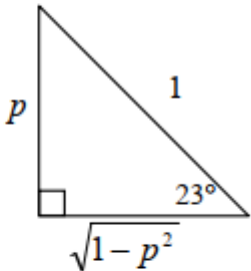
QUESTION 8

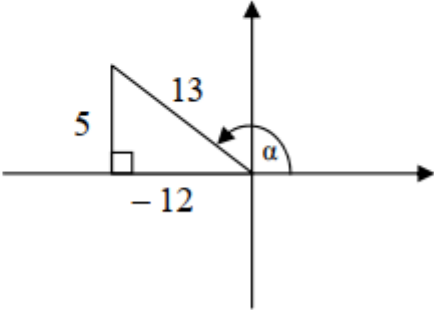
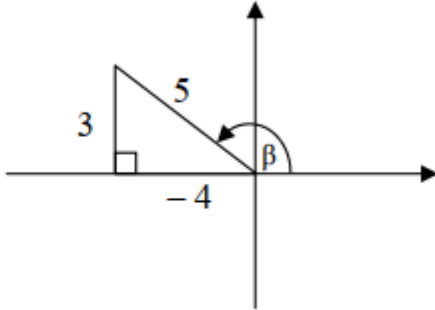


8.2	$\sin 2x = \frac{1}{2} \tan x$ $2 \sin x \cdot \cos x = \frac{\sin x}{2 \cos x}$ $4 \sin x \cdot \cos^2 x - \sin x = 0$ $\sin x (4 \cos^2 x - 1) = 0$ $\sin x = 0 \qquad \cos^2 x = \frac{1}{4}$ $x = 0^\circ \text{ or } 180^\circ \text{ or } \cos x = \pm \frac{1}{2}$ $x = 60^\circ ; -60^\circ \text{ or } 120^\circ$
8.3	$\{x \mid -60^\circ < x < 0^\circ\} \cup \{x \mid 60^\circ < x < 90^\circ\} \cup \{x \mid 120^\circ < x < 180^\circ\}$ OR $x \in (-60^\circ ; 0^\circ) \cup (60^\circ ; 90^\circ) \cup (120^\circ ; 180^\circ)$ OR $-60^\circ < x < 0^\circ \text{ or } 60^\circ < x < 90^\circ \text{ or } 120^\circ < x < 180^\circ$

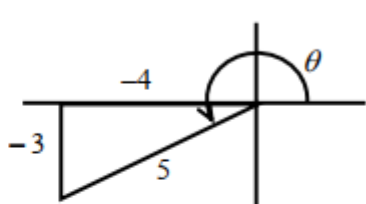
PAPER F

QUESTION 6

6.1.1	$\cos 113^\circ$ $= \cos (90^\circ + 23^\circ)$ $= -\sin 23^\circ$ $= -p$
6.1.2	$\cos 23^\circ$ $= \sqrt{1 - p^2}$  OR $\cos^2 23^\circ + \sin^2 23^\circ = 1$ $\cos^2 23^\circ = 1 - p^2$ $\cos 23^\circ = \sqrt{1 - p^2}$

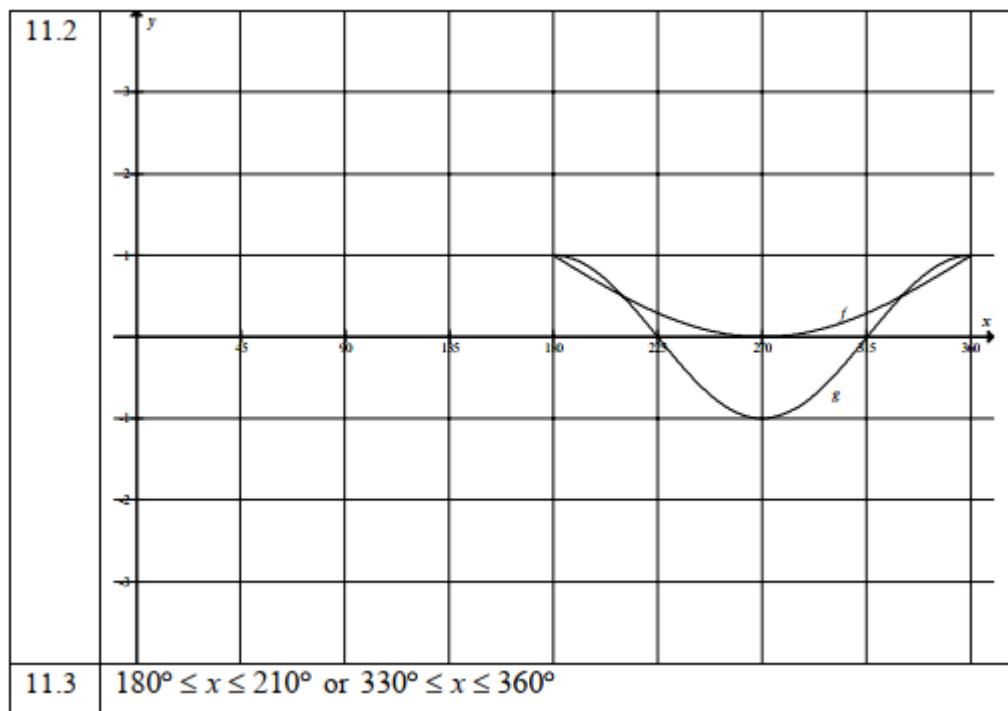
6.1.3	$\sin 46^\circ$ $= 2 \sin 23^\circ \cdot \cos 23^\circ$ $= 2p\sqrt{1-p^2}$
6.2.1	$\sin \alpha = \frac{5}{13}$ $y_\alpha = 5 \quad r_\alpha = 13$ $x_\alpha = -12$ $\cos \alpha = -\frac{12}{13}$ 
6.2.2	$\tan \beta = -\frac{3}{4}$ $y_\beta = 3 \quad x_\beta = -4$ $r = 5$  $\cos(\alpha + \beta)$ $= \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$ $= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$ $= \frac{48-15}{65}$ $= \frac{33}{65}$
6.3	$\frac{1}{2} \cos x = 0,435$ $\cos x = 0,87$ $x = 29,54^\circ \quad \text{or} \quad x = 330,46^\circ$

QUESTION 9

9.1.1	$\sin \theta = -\frac{3}{5} \text{ and } \cos \theta = -\frac{4}{5}$ $\sin \theta + \cos \theta = -\frac{7}{5}$	
9.1.2	$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$ $= \frac{2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)}{\frac{16}{25} - \frac{9}{25}}$ $= \frac{24}{7}$	
9.2.1	$\frac{\cos(360^\circ - x) \cdot \tan^2 x}{\sin(x - 180^\circ) \cdot \cos(90^\circ + x)}$ $= \frac{(\cos x)(\tan^2 x)}{(-\sin x)(-\sin x)}$ $= (\cos x) \left(\frac{\sin^2 x}{\cos^2 x} \right) \left(\frac{1}{\sin^2 x} \right)$ $= \frac{1}{\cos x}$	
9.2.2	$x = 30^\circ$ $\frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$	

QUESTION 11

11.1	$1 + \sin x = \cos 2x$ $1 + \sin x = 1 - 2 \sin^2 x$ $\sin x + 2 \sin^2 x = 0$ $\sin x(1 + 2 \sin x) = 0$ $\sin x = 0 \quad \text{or} \quad \sin x = -\frac{1}{2}$ $x = k \cdot 180 \quad \text{or} \quad x = -30^\circ + k \cdot 360 \quad k \in \mathbb{Z}$ $x = 210^\circ + k \cdot 360$ $x \in \{180^\circ; 210^\circ; 330^\circ; 360^\circ\}$
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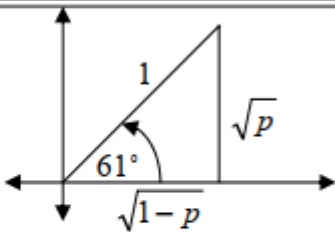
PAPER G

QUESTION 10

10.1.1	$\cos 28^\circ = \sqrt{1 - \sin^2 28^\circ}$ $= \sqrt{1 - a^2}$
10.1.2	$\cos 64^\circ$ $= \cos 2(32^\circ)$ $= 2 \cos^2 32^\circ - 1$ $= 2b^2 - 1$
10.1.3	$\sin 4^\circ$ $= \sin(32^\circ - 28^\circ)$ $= \sin 32^\circ \cos 28^\circ - \cos 32^\circ \sin 28^\circ$ $= \sqrt{1 - b^2} \cdot \sqrt{1 - a^2} - ab$

10.2	$ \begin{aligned} & b\sqrt{1-a^2} - a\sqrt{1-b^2} \\ &= \cos 32^\circ \cdot \sqrt{1 - \sin^2 28^\circ} - \sin 28^\circ \sqrt{1 - \cos^2 32^\circ} \\ &= \cos 32^\circ \cdot \cos 28^\circ - \sin 28^\circ \cdot \sin 32^\circ \\ &= \cos(32^\circ + 28^\circ) \\ &= \cos 60^\circ \\ &= \frac{1}{2} \end{aligned} $
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QUESTION 11

11.1.1	$ \begin{aligned} \sin 61^\circ &= \sqrt{p} \\ \sin 241^\circ &= \sin(180^\circ + 61^\circ) \\ &= -\sin 61^\circ \\ &= -\sqrt{p} \end{aligned} $	
11.1.2	$ \begin{aligned} \cos 61^\circ &= \sqrt{1 - \sin^2 61^\circ} \\ &= \sqrt{1 - p} \end{aligned} $	
11.1.3	$ \begin{aligned} \cos 122^\circ &= \cos 2(61^\circ) \\ &= 2\cos^2 61^\circ - 1 \\ &= 2(\sqrt{1-p})^2 - 1 \\ &= 2(1-p) - 1 \\ &= 2 - 2p - 1 \\ &= 1 - 2p \end{aligned} $	
11.1.4	$ \begin{aligned} &\cos 73^\circ \cos 15^\circ + \sin 73^\circ \sin 15^\circ \\ &= \cos(73^\circ - 15^\circ) \\ &= \cos 58^\circ = (\cos 180^\circ - 122^\circ) \\ &= -(\cos 122^\circ) \\ &= -(1 - 2p) \\ &= 2p - 1 \end{aligned} $	

11.2.1	$\begin{aligned} \text{LHS} &= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)} \\ &= \frac{\cos^2 x + 2 \cos x \sin x + \sin^2 x - (\cos^2 x - 2 \sin x \cos x + \sin^2 x)}{(\cos x - \sin x)(\cos x + \sin x)} \\ &= \frac{4 \cos x \sin x}{\cos^2 x - \sin^2 x} \\ &= \frac{2 \sin 2x}{\cos 2x} \\ &= 2 \tan x \\ &= \text{RHS} \end{aligned}$
11.2.2	$\cos x = \sin x \quad \text{or} \quad \cos x = -\sin x$ $x = 45^\circ \quad \quad \quad x = 135^\circ$
11.3.1	$\sin x = \cos 2x - 1$ $\sin x = 1 - 2 \sin^2 x - 1$ $\sin x = -2 \sin^2 x$ $2 \sin^2 x + \sin x = 0$
11.3.2	$\sin x = \cos 2x - 1$ $2 \sin^2 x + \sin x = 0$ $\sin x (2 \sin x + 1) = 0$ $\sin x = 0 \quad \text{or} \quad \sin x = -\frac{1}{2}$ $\therefore x = 0^\circ + 180^\circ k, \quad k \in \mathbb{Z} \quad \text{or} \quad x = \{210^\circ \text{ or } 330^\circ\} + 360^\circ k, \quad k \in \mathbb{Z}$ OR $x = n \cdot 180^\circ$ $x = n \cdot 360^\circ - 30^\circ$ $x = (2n+1) \cdot 180^\circ + 30^\circ, n \in \mathbb{Z}$
11.4	$\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \tan 4^\circ \times \dots \times \tan 87^\circ \times \tan 88^\circ \times \tan 89^\circ$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ} \right) \left(\frac{\sin 2^\circ}{\cos 2^\circ} \right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ} \right) \dots \left(\frac{\sin 88^\circ}{\cos 88^\circ} \right) \left(\frac{\sin 89^\circ}{\cos 89^\circ} \right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ} \right) \left(\frac{\sin 2^\circ}{\cos 2^\circ} \right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ} \right) \dots \left(\frac{\sin(90^\circ - 2^\circ)}{\cos(90^\circ - 2^\circ)} \right) \left(\frac{\sin(90^\circ - 1^\circ)}{\cos(90^\circ - 1^\circ)} \right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ} \right) \left(\frac{\sin 2^\circ}{\cos 2^\circ} \right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ} \right) \dots \left(\frac{\cos 2^\circ}{\sin 2^\circ} \right) \left(\frac{\cos 1^\circ}{\sin 1^\circ} \right)$ $= \tan 45^\circ$ $= 1$

EUCLID'S GEOMETRY

PAPER A

A1

QUESTION 7

7.1.1 equal to twice the angle subtended by the same chord at the circle.

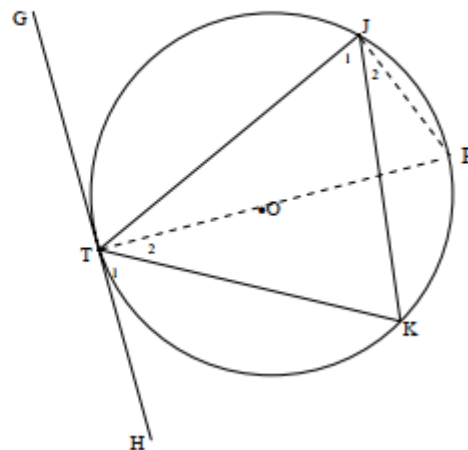
7.1.2 equal to the angle subtended by the same chord in the alternate segment.

7.1.3 supplementary.

A2

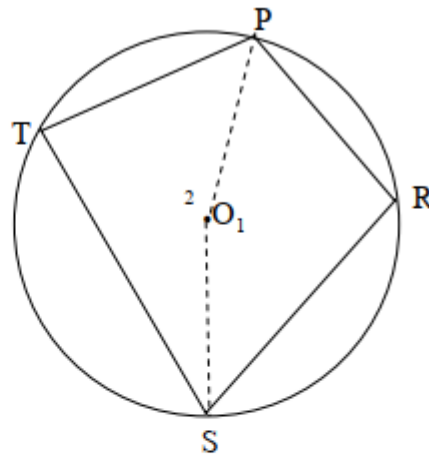
QUESTION 8

- 8.1 Draw diameter TP.
Join P to J.
 $\hat{T}_1 + \hat{T}_2 = 90^\circ$ (tan $\perp$ diameter)
 $\hat{J}_1 + \hat{J}_2 = 90^\circ$ ($\angle$ in semi-circle)
 $\hat{J}_2 = \hat{T}_2$ ($\angle$ in same seg)
 $\hat{T}_1 = \hat{J}_1$



A3

QUESTION 9



9.1 Join PO and OS

$$\text{Let } \hat{O}_1 = 2x$$

$$\hat{T} = x \quad (\angle \text{ at circ centre} = 2 \angle \text{ at circumference})$$

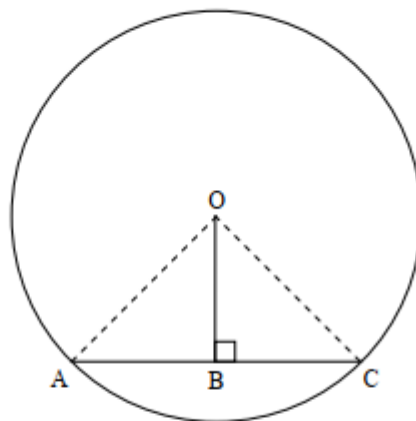
$$\hat{O}_2 = 360^\circ - 2x \quad (\angle \text{ s round a point})$$

$$\hat{R} = 180^\circ - x \quad (\angle \text{ at circ centre} = 2 \angle \text{ at circumference})$$

$$\begin{aligned} \hat{T} + \hat{R} &= x + 180^\circ - x \\ &= 180^\circ \end{aligned}$$

A4

QUESTION/VRAAG 11



Construct radii OA and OC.

In $\triangle OAB$ and $\triangle OCB$

i. OB is common

ii. OA = OC (radii)

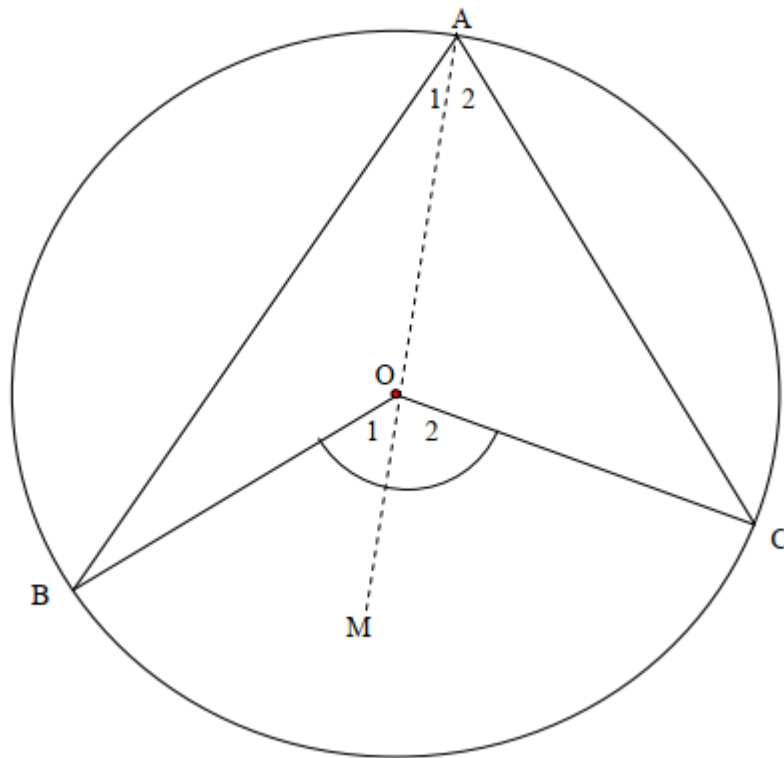
iii. $\hat{OBA} = \hat{OBC} = 90^\circ$ (given)

$\triangle OAB \equiv \triangle OCB$ (90°HS)

AB = BC ($\equiv \Delta$ s)

A5

QUESTION/VRAAG 10

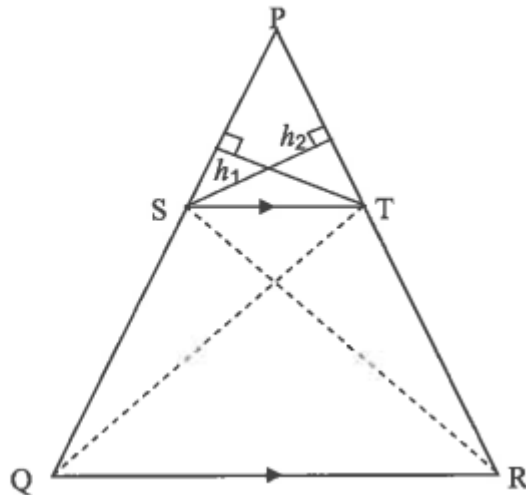


10.1	Construction: AO is drawn and produced to M $\hat{O}_1 = \hat{A}_1 + \hat{B} \quad [\text{ext } \angle \text{ of } \Delta / \text{buite } \angle \text{ van } \Delta]$ But $\hat{A}_1 = \hat{B} \quad [\angle \text{s opp} = \text{radii} / \angle \text{e teenoor} = \text{radii}]$ $\therefore \hat{O}_1 = 2\hat{A}_1$ Similarly/Netso: $\hat{O}_2 = 2\hat{A}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$ $= 2(\hat{A}_1 + \hat{A}_2)$ $\hat{BOC} = 2\hat{BAC}$
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A6

QUESTION/VRAAG 10

10.1



10.1

Constr : Join S to R and T to Q and draw h_1 from $S \perp PT$ and h_2 from $T \perp PS$ / Verbind SR en TQ en trek h_1 van $S \perp PT$ en h_2 van $T \perp PS$

Proof :

$$\frac{\text{area } \triangle PST}{\text{area } \triangle QST} = \frac{\frac{1}{2} PS \times h_2}{\frac{1}{2} SQ \times h_2} = \frac{PS}{SQ} \quad \text{equal altitudes}$$

$$\frac{\text{area } \triangle PST}{\text{area } \triangle STR} = \frac{\frac{1}{2} PT \times h_1}{\frac{1}{2} TR \times h_1} = \frac{PT}{TR} \quad \text{equal altitudes}$$

$$\text{area } \triangle PST = \text{area } \triangle PST \quad [\text{common}]$$

$$\text{But area } \triangle QST = \text{area } \triangle STR \quad [\text{same base, height; } ST \parallel QR]$$

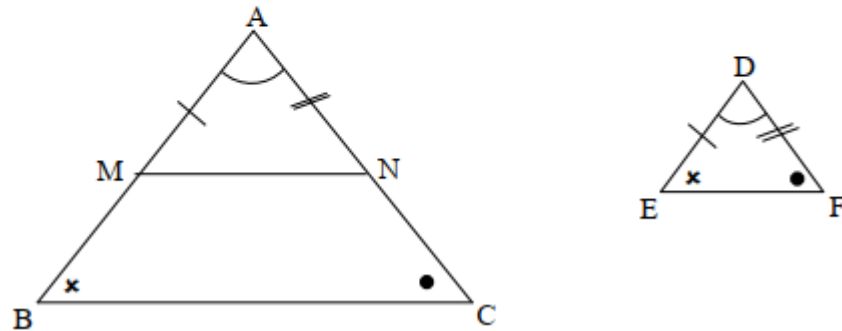
$$\therefore \frac{\text{area } \triangle PST}{\text{area } \triangle QST} = \frac{\text{area } \triangle PST}{\text{area } \triangle STR}$$

$$\therefore \frac{PS}{SQ} = \frac{PT}{TR}$$

A7

QUESTION/VRAAG 10

10.1

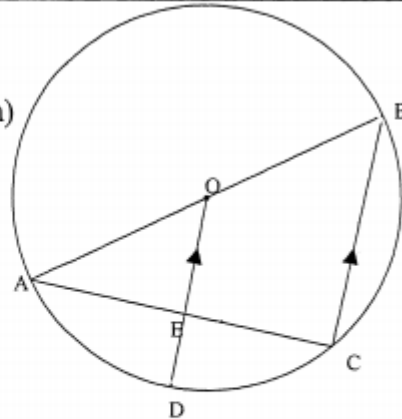


10.1	Constr: Let M and N lie on AB and AC respectively such that $AM = DE$ and $AN = DF$. Draw MN. Konst: <i>Merk M en N op AB en AC onderskeidelik af sodanig dat $AM = DE$ en $AN = DF$. Verbind MN.</i> Proof: In $\triangle AMN$ and $\triangle DEF$ $AM = DE$ [Constr] $AN = DF$ [Constr] $\hat{A} = \hat{D}$ [Given] $\therefore \triangle AMN \cong \triangle DEF$ (SAS) $\therefore \hat{AMN} = \hat{E} = \hat{B}$ $MN \parallel BC$ [corresp $\angle$'s are equal/ooreenkomstige $\angle$e =] $\frac{AB}{AM} = \frac{AC}{AN}$ [line $\parallel$ one side of $\triangle$ OR prop theorem; $MN \parallel BC$] $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [$AM = DE$ and $AN = DF$]
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PAPER B

QUESTION 9

9. $\hat{C} = 90^\circ$ ($\angle$ s in semi circle)
 $\hat{OEA} = 90^\circ$ (corres $\angle$ s; $OD \parallel BC$)
 $AE = 8$ cm (line from circ cent $\perp$ ch bis ch)
 $OE = 6$ cm (Pythagoras)
 $ED = 10 - 6$
 $= 4$ cm



OR
 $\hat{C} = 90^\circ$ ($\angle$ s in semi circle)
 $\hat{OEA} = 90^\circ$ (corres $\angle$ s; $OD \parallel BC$)
 $OE \parallel BC$ (given)
 $OA = OB$ (radii)
 $AE = EC = 8$ cm (midpoint theorem)
 $OE = 6$ cm (Pythagoras)
 $ED = 10 - 6$
 $= 4$ cm

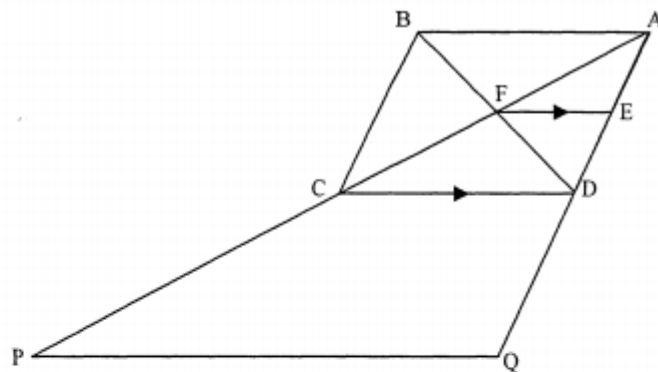
OR
 $\hat{C} = 90^\circ$ ($\angle$ s in semi circle)
 $BC^2 = (20)^2 - (16)^2$
 $BC^2 = 144$
 $BC = 12$
 $OE = \frac{1}{2} BC$ (midpoint theorem)
 $OE = 6$ cm
 $OD = 10$ cm
 $ED = 10 - 6$
 $= 4$ cm

OR
 $\hat{C} = 90^\circ$ ($\angle$ s in semi circle)
 $BC^2 = (20)^2 - (16)^2$
 $BC^2 = 144$
 $BC = 12$
 $OE = \frac{1}{2} BC$ (midpoint theorem)
 $OE = 6$ cm
 $ED = 4$ cm

Question 10

10.1	$\hat{A} = \hat{D}_4 = x$ (tan ch th) $\hat{E}_2 = x$ (tan ch th) OR ($\angle$ s in same seg) $\hat{D}_2 = \hat{A} = x$ (alt $\angle$ s; CA $\parallel$ DF)
10.2	In $\triangle BHD$ and $\triangle FED$ 1. $\hat{B}_2 = \hat{F}$ ($\angle$ s in same seg) 2. $\hat{D}_3 = \hat{D}_1$ (= chs subt = $\angle$ s) $\triangle BHD \parallel \triangle FED$ ($\angle\angle\angle$)
10.3	$\frac{FE}{BH} = \frac{FD}{BD}$ ($\parallel \triangle$ s) But $FE = AB$ (given) $\frac{AB}{BH} = \frac{FD}{BD}$ $AB \cdot BD = FD \cdot BH$

QUESTION 11



11.1	$AF = FC$ $FE \parallel CD$ $AE = ED$	(diags of parallelogram bisect) (Prop Th; $FE \parallel CD$) OR (Midpoint Theorem)
11.2	$\frac{AC}{CP} = \frac{1}{2}$ $\frac{AD}{DQ} = \frac{1}{2}$ $\frac{AC}{CP} = \frac{AD}{DQ}$ $CD \parallel PQ$ $CD \parallel FE$ $\therefore PQ \parallel FE$	(given) (given) (converse proportionality theorem) (given)

OR

$$\frac{AC}{AP} = \frac{1}{3}$$

$$\frac{AD}{AQ} = \frac{1}{3}$$

$$\frac{AC}{AP} = \frac{AD}{AQ}$$

$CD \parallel PQ$ (converse proportionality theorem)

$CD \parallel FE$ (given)

$\therefore PQ \parallel FE$

OR

$$\frac{AF}{AP} = \frac{1}{6}$$

$$\frac{AE}{AQ} = \frac{1}{6}$$

$$\frac{AF}{AP} = \frac{AE}{AQ}$$

$\therefore PQ \parallel FE$ (converse proportionality theorem)

11.3

In $\triangle AEF$ and $\triangle APQ$

1. $\hat{A}$ is common

2. $\hat{AEF} = \hat{AQP}$ (corres $\angle$ s; $FE \parallel PQ$)

3. $\hat{AFE} = \hat{APQ}$ (corres $\angle$ s; $FE \parallel PQ$)

$\therefore \triangle AEF \sim \triangle AQP$ ($\angle\angle\angle$)

$$\frac{FE}{PQ} = \frac{AF}{AP} \quad (\sim \triangle s)$$

$$\frac{FE}{60} = \frac{1}{6}$$

$$FE = 10 \text{ cm}$$

OR

In $\triangle ADC$ and $\triangle APQ$

1. $\hat{A}$ is common
2. $\hat{ADC} = \hat{AQP}$ (corres $\angle$ s; $CD \parallel PQ$)
3. $\hat{ACD} = \hat{APQ}$ (corres $\angle$ s; $CD \parallel PQ$)

$\therefore \triangle ADC \parallel \triangle AQP$ ($\angle\angle\angle$)

$$\frac{AC}{AP} = \frac{AD}{AQ} = \frac{1}{3} \quad (\parallel \Delta s)$$

$$CD = \frac{1}{3} PQ$$

$$CD = 20 \text{ cm}$$

But $AF = FC$

$$AE = ED \quad (\text{Midpoint Theorem})$$

$$FE = \frac{1}{2} CD$$

$$FE = 10 \text{ cm}$$

PAPER C

9.2.1	$\frac{EM}{AM} = \frac{FD}{AD}$ $\frac{EM}{AM} = \frac{3}{7}$	(Line parallel one side of $\triangle$) OR prop th; $EF \parallel BD$) (<i>Lyn ewewydig aan sy v $\triangle$</i>) OF eweredigst; $EF \parallel BD$)
9.2.2	$CM = AM$ $\frac{CM}{ME} = \frac{AM}{ME} = \frac{7}{3}$	(diags of parm bisect/<i>hoekl parm halv</i>) (from 9.2.1/<i>vanaf 9.2.1</i>)
9.2.3	$h \text{ of } \triangle FDC = h \text{ of } \triangle BDC$ $\frac{\text{area } \triangle FDC}{\text{area } \triangle BDC} = \frac{\frac{1}{2} FD \cdot h}{\frac{1}{2} BC \cdot h}$ $= \frac{FD}{AD}$ $= \frac{3}{7}$	($AD \parallel BC$) (opp sides of parm =) (<i>tos sye v parm =</i>)

PAPER D

8.1.1	$\hat{D}_2 + \hat{D}_3 = 90^\circ$ (tan $\perp$ radius) $\therefore \hat{D}_2 = 90^\circ - 40^\circ = 50^\circ$
8.1.2	$\hat{D}_1 = \hat{C}_2 = 65^\circ$ (angles opposite equal sides) $\hat{D}_1 + \hat{D}_2 = \hat{FBC}$ (ext angle of a cyclic quad) $65^\circ + 50^\circ = \hat{FBC}$ $\therefore \hat{FBC} = 115^\circ$

8.2.1	$\hat{S} = 32^\circ$ ($\angle$ s in the same segment)
8.2.2	$\hat{O}_2 = 64^\circ$ ($\angle$ at centre = $2 \times \angle$ at circumference)
8.2.3	$\hat{T}_1 = 90^\circ$ (line from centre to midpoint of chord) $\hat{O}_1 + \hat{T}_1 + \hat{M}_2 = 180^\circ$ (sum of angles of Δ) $\hat{O}_1 + 90^\circ + 15^\circ = 180^\circ$ $\hat{O}_1 = 75^\circ$

QUESTION 9

9.1	In ΔMBC: $\hat{B}_2 = \hat{B}_3 = x$ BE bisects $\hat{MBC}$ $\therefore \hat{MBC} = 2x$ $\hat{MBC} = \hat{MCB} = 2x$ angles opposite equal sides In ΔBEC: $\hat{E}_2 = 180^\circ - (x+x)$ Sum of angles of a Δ $= 180^\circ - 2x$
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9.2	In $\triangle BMC$: $\hat{BMC} = 180^\circ - (2x + 2x)$ Sum of angles of a $\triangle$ $= 180^\circ - 4x$ But $\hat{BAC} = \frac{1}{2}\hat{BMC}$ $\angle$ at centre twice angle $= \frac{1}{2}(180^\circ - 4x)$ $= 90 - 2x$
9.3	In $\triangle ABE$: $\hat{E}_1 + \hat{E}_2 = 180^\circ$ Straight line $\hat{E}_1 = 180^\circ - \hat{E}_2$ $= 180^\circ - (180^\circ - 2x)$ $= 2x$ In $\triangle ABE$: $\hat{ABE} + \hat{BAC} + \hat{E} = 180^\circ$ Sum of $\angle$ s of $\triangle$ $\hat{ABE} = 180^\circ - (\hat{BAC} + \hat{E}_1)$ $= 180^\circ - (90^\circ - 2x + 2x)$ $= 90^\circ$ $\therefore$ AE is a diameter of circle ABE (Subtends) $\angle 90^\circ$

QUESTION 10

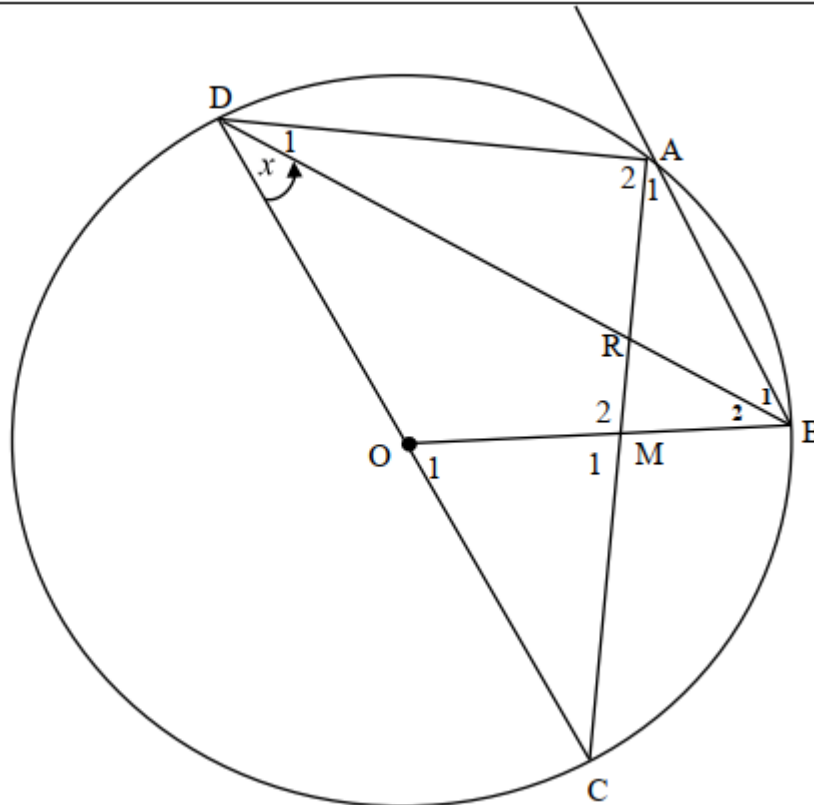
10.1.1	Let $\hat{Y}_1 = a$ and $\hat{N} = b$ $\therefore \hat{T}_3 = a - b$ (ext. $\angle$ of Δ = sum opp. $\angle$s) $\hat{T}_1 = \hat{N} = b$ (tan XT; chord MT) $X\hat{T}Y = a$ (angles opposite equal sides) $\hat{T}_2 = X\hat{T}Y - \hat{T}_1$ $= a - b$ $\therefore \hat{T}_3 = \hat{T}_2$ $\therefore YT$ bisects $M\hat{T}N$
10.1.2	In ΔXMT and ΔXTN: $\hat{X}$ is common $\hat{T}_1 = \hat{N}$ tan XT; chord MT $\hat{M}_1 = X\hat{T}N$ remaining $\angle$ $\therefore \Delta XMT \parallel \Delta XTN$ $\angle\angle\angle$ $\therefore \frac{XM}{XT} = \frac{XT}{XN} = \frac{MT}{TN}$ similar Δ's $\therefore \frac{XM}{XT} = \frac{XT}{XN}$
10.2.1	$XM = XY - 20$ $XY = XT$ $= k - 20$
10.2.2	$\frac{XM}{XT} = \frac{XT}{XN}$ $\therefore \frac{k - 20}{k} = \frac{k}{k + 50}$ $\therefore (k - 20)(k + 50) = k^2$ $\therefore k^2 + 30k - 1000 = k^2$ $\therefore 30k - 1000 = 0$ $\therefore 30k = 1000$ $\therefore k = 33,3 \text{ mm}$

PAPER E

8.1.1	$\hat{A}\hat{D}\hat{C} = 67^\circ$ OR / OF $\hat{B}_2 + \hat{B}_3 = 113^\circ$ $\hat{A}\hat{D}\hat{C} = 67^\circ$	ext. $\angle$ of cyclic quad / buite $\angle$ van kvh $\angle^s$ straight line / $\angle^e$ op reguit lyn opp $\angle^s$ of cyclic quad / oorst $\angle^e$ van kvh
8.1.2	$\hat{C} = 180^\circ - 67^\circ$ $= 113^\circ$	co-int $\angle^s$ $BC \parallel AD$ / ko-binne $\angle^e$ $BC \parallel AD$
8.1.3	$\hat{A} = 67^\circ$	opp $\angle^s$ of cyclic quad / alt $\angle^s$ $BC \parallel AD$ / alt $\angle^s$ $EC \parallel AD$ oorst $\angle^e$ van kvh / verwisselende $\angle^e$ $BC \parallel AD$ / verwis $\angle^e$ $EC \parallel AD$
8.1.4	$\hat{B}_2 = 67^\circ$ $\hat{D}_2 = 180^\circ - 67^\circ - 67^\circ$ $= 46^\circ$	$\angle^s$ opposite = sides / $\angle^e$ teenoor = sye sum of $\angle^s$ in Δ / som vd $\angle^e$ v Δ
8.1.5	$\hat{B}\hat{D}\hat{G} = 113^\circ$ OR / OF $\hat{D}_1 = 67^\circ$ $\hat{B}\hat{D}\hat{G} = 113^\circ$	tan chord theorem / raaklyn koordstelling tan chord theorem / raaklyn koordstelling
8.2	$\hat{B}_3 = \hat{D}_2 = 46^\circ$ $AB = CD$	alt $\angle^s$ $BC \parallel AD$ / verwisselende $\angle^e$ $BC \parallel AD$ $\angle^s$ subtend = chords / $\angle^e$ onderspan = koorde

QUESTION / VRAAG 9

9.1



9.1.1(a)

$$\hat{O}_1 = 2x$$

$\angle \text{centre} = 2 \times \angle \text{circumference}$ /
middelpunts $\angle = 2 \times \text{omtreks } \angle$

9.1.1(b)

$$\hat{A}_1 = \hat{CDB} = x$$

$\angle^s$ in the same segment /

$\angle^e$ in dies. segment

$$\hat{M}_2 = 90^\circ$$

line from centre to midpoint of chord / *lyn*
van middelpunt van sirkel na middelpunt
van koord

$$\therefore \hat{ABO} = 90^\circ - x$$

sum of $\angle^s$ in Δ / ext $\angle$ of a Δ /
som vd $\angle^e$ v Δ / buite $\angle$ v Δ

OR/OF

$$\hat{O}_1 = 2x$$

proved/reeds bewys

$$\hat{M}_1 = 90^\circ$$

line from centre to midpoint of chord / *lyn*
van middelpunt van sirkel na middelpunt
van koord

$$\hat{C} = 90^\circ - 2x$$

sum of $\angle^s$ in Δ / *som vd $\angle^e$ v Δ*

$$\hat{B}_1 = 90^\circ - 2x$$

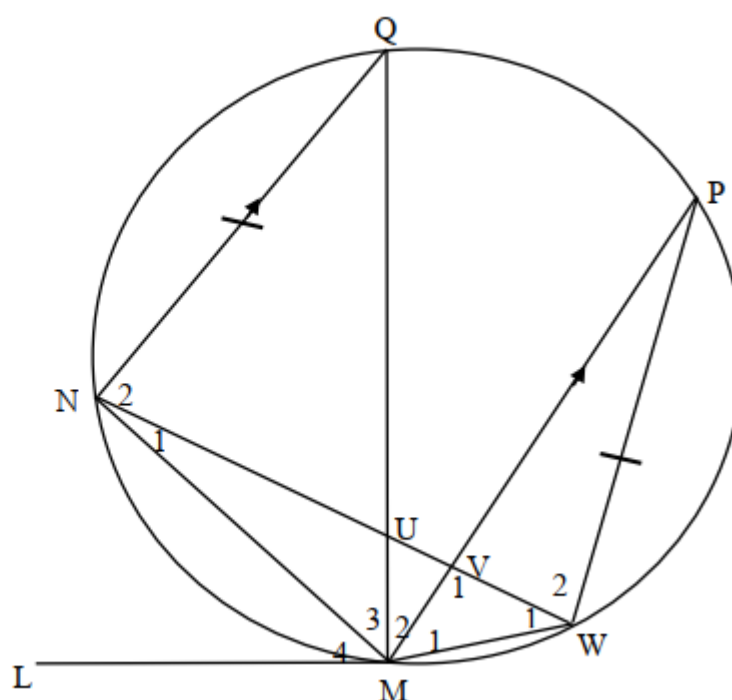
$\angle^s$ in the same segment /

$\angle^e$ in dies. segment

$$\hat{ABO} = 90^\circ - x$$

9.1.2	$AD \parallel OB$ $\hat{O}_1 = \hat{ADC} = 2x$ $\therefore \hat{D}_1 = x$ $\hat{A}_1 = x$ $\therefore \hat{D}_1 = \hat{A}_1$ AB is a tangent / is 'n raaklyn OR / OF $\hat{A}_2 = 90^\circ$ $\therefore AD \parallel OB$ $\hat{CDA} = \hat{O}_1 = 2x$ $\therefore \hat{D}_1 = x$ $\hat{D}_1 = \hat{A}_1$ AB is a tangent / is 'n raaklyn	midpoint theorem / middelpunt stelling corresponding $\angle^s$ $AD \parallel OB$ / ooreenkom $\angle^s$ $AD \parallel OB$ proved / reeds bewys converse tan chord theorem / omgekeerde raaklyn koordstelling $\angle$ in a semi-circle / $\angle$ in halwe sirkel corr $\angle^s$ are equal / ooreenk $\angle^s$ gelyk corr $\angle^s$ $DA \parallel OB$ / ooreenk $\angle^s$ $DA \parallel OB$ converse tan chord theorem / omgekeerde raaklyn koord
9.1.3	$DC^2 = AD^2 + AC^2$ but / maar $AC = 2AM$ and / en $DC = 2DO$ $(2DO)^2 = AD^2 + (2AM)^2$ $4DO^2 = AD^2 + 4AM^2$ but / maar In $\triangle ABM$ $AM^2 = AB^2 - MB^2$ $\therefore 4DO^2 = AD^2 + 4(AB^2 - MB^2)$ $AD^2 = 4DO^2 - 4AB^2 + 4MB^2$	Pythagoras Pythagoras

9.2



9.2.1

$$\hat{M}_2 = \hat{Q}$$

alternate $\angle^s$ $NQ \parallel MP$ /

verwissel $\angle^s$ $NQ \parallel MP$

$$\hat{M}_4 = \hat{Q} = \hat{M}_2$$

tan chord theorem / raaklyn koordstelling

$$\hat{W}_1 = \hat{Q} = \hat{M}_2$$

 $\angle^s$ in the same segment / tan chord

theorem / $\angle^s$ dieselfde sirkel segment /
raaklyn koord stelling

OR/OF

$$\hat{W}_1 = \hat{M}_4 = \hat{M}_2$$

tan chord theorem / raaklyn
koordstelling

9.2.2

In $\triangle WMV$ and/en $\triangle QMN$

$$\hat{W}_1 = \hat{Q}$$

proved / reeds bewys

$$\hat{M}_1 = \hat{M}_3$$

equal chords subtend equal $\angle^s$ / gelyke
koorde onderspan gelyke $\angle^s$

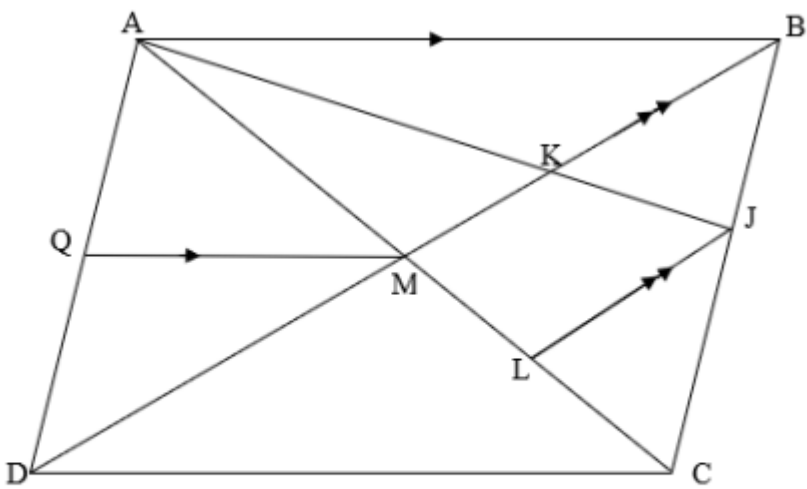
$$\hat{V}_1 = \hat{N}_1 + \hat{N}_2$$

sum of $\angle^s$ in $\triangle$ / som vd $\angle^s$ v $\triangle$

$$\therefore \triangle WMV \parallel \triangle QMN$$

 $\angle \angle \angle$

9.2.3	$\frac{MV}{MN} = \frac{WV}{QN}$ $\frac{MV}{WV} = \frac{MN}{QN}$ $MV \times QN = MN \times WV$ $\text{but / maar } QN = PW$ $MV \times PW = MN \times WV$ $\frac{MV}{WV} = \frac{MN}{PW}$	$\triangle WMV \parallel \triangle QMN \parallel \triangle^s$ given / gegee
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10.2		
10.2.1 (a)	$\frac{ML}{LC} = \frac{BJ}{JC} = \frac{2}{3}$	line $\parallel$ one side $\triangle BCM$ OR prop theorem $MB \parallel JL$ / lyn $\parallel$ aan een sy van $\triangle BCM$ OF eweredigheidsstelling $MB \parallel JL$
10.2.1 (b)	$\frac{MC}{ML} = \frac{BC}{BJ} = \frac{5}{2}$ $AM = MC$ $\frac{AM}{ML} = \frac{5}{2}$ $\frac{AK}{KJ} = \frac{AM}{ML} = \frac{5}{2}$	line $\parallel$ one side $\triangle BMC$ OR prop theorem $MB \parallel JL$ / lyn $\parallel$ aan een sy van $\triangle BMC$ OF eweredigheidsstelling $MB \parallel JL$ diagonals of a parm bisect / hoeklyne van parm halveer line $\parallel$ one side $\triangle AJL$ OR prop theorem $MK \parallel JL$ / lyn $\parallel$ aan een sy van $\triangle AJL$ OF eweredigheidsstelling $MK \parallel JL$

10.2.2	$AB \parallel CD$ $AB \parallel QM$ In $\triangle ADC$ $\therefore QM \parallel CD$ $AM = MC$ $\therefore AQ = QD$ <i>but</i> $AD = BC$ $AQ = \frac{1}{2} AD$ $= \frac{1}{2} \left(\frac{2\sqrt{10}}{3} \right)$ $\therefore AQ = QD = \frac{2}{3} \sqrt{10} \div 2$ $= \frac{\sqrt{10}}{3} \text{ units}$	opposite sides of parm / oorst sye van parm proved / reeds bewys line passing through the midpoint of 1 side $\parallel$ to second side / lyn sny die middelpunt van 1 sy $\parallel$ aan tweede sy opposite sides of parm / oorst sye van parm
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PAPER F

8.1	$\hat{O}_2 = 50^\circ$ $\hat{D}_1 = 25^\circ$	$\angle$ s around a point / $\angle$ e om 'n punt $\angle$ centre = $2 \times \angle$ at circumference midpts $\angle = 2 \times$ omtreks $\angle$
8.2	$\hat{B}_3 = 25^\circ$	tan chord theorem / raaklyn koordstelling
8.3	$\hat{BCD} = 120^\circ$ $\hat{B}_2 = 35^\circ$ $\hat{OBC} = \hat{OCB} = 65^\circ$ $\therefore \hat{B}_1 = 65^\circ - 35^\circ$ $\hat{B}_1 = 30^\circ$ OR / OF $\hat{BCD} = 120^\circ$ $\hat{B}_2 = 35^\circ$ $\hat{B}_1 + \hat{B}_2 + \hat{B}_3 = 90^\circ$ $\hat{B}_1 = 30^\circ$	opp $\angle$ s of a cyclic quad / teenoorst $\angle$ e v kvl sum of $\angle$ s of a triangle / som $\angle$ e v $\triangle$ $\angle$ s opp. equal radii / $\angle$ e teenoor gelyke radiuse OR / OF opp $\angle$ s of a cyclic quad / teenoorst $\angle$ e v kvh sum of $\angle$ s of a triangle / som $\angle$ e v $\triangle$ radius $\perp$ tangent / radius $\perp$ raaklyn

QUESTION / VRAAG 9

9.1	Equal / gelyk.	
9.2.1	$\hat{D}_1 = \hat{F}_1 = x$ $\hat{F}_1 = \hat{F}_2 = x$	tan chord theorem / raaklyn koordstelling = chords subtend = $\angle s$ = koorde onderspan = $\angle e$
9.2.2	$\hat{F}_2 = \hat{A} = x$ $\hat{D}_1 = \hat{A} = x$ ABDC is a cyclic quad / ABDC is 'n kvh	ext. $\angle$ of cyclic quad / buite $\angle$ v kvh ext $\angle$ = opp int $\angle$ OR converse of ext. $\angle$ of cyclic quad / buite $\angle$ = oorst binne $\angle$ OF omgekeerde buite $\angle$ v kvh
9.2.3	$\hat{B}_1 + \hat{B}_2 = \hat{A}$ $\hat{A} = \hat{D}_1$ $\hat{B}_1 + \hat{B}_2 = \hat{D}_1$ BE $\parallel$ CD	tan chord theorem / raaklyn koordstelling proved / reeds bewys correspond $\angle s$ = / ooreenkomst $\angle e$ =
9.2.4	$\hat{C}_1 + \hat{C}_2 + \hat{F}_1 + \hat{F}_2 = 180^\circ$ $\hat{C}_1 = \hat{C}_2$ $\hat{F}_1 = \hat{F}_2$ $2\hat{C}_1 + 2\hat{F}_2 = 180^\circ$ $\hat{C}_1 + \hat{F}_2 = 90^\circ$ $\hat{E}_1 = 90^\circ$ FC is a diameter of circle FDCE. FC is 'n middellyn van sirkel FDCE.	opp $\angle s$ of a cyclic quad / teenoorst $\angle e$ v kvh diag rhombus bisect $\angle$ / diag ruit halveer $\angle$ proved / reeds bewys sum of $\angle s$ of Δ / som van $\angle e$ v Δ converse $\angle$ in a semi circle / omgekeerde $\angle$ in half sirkel

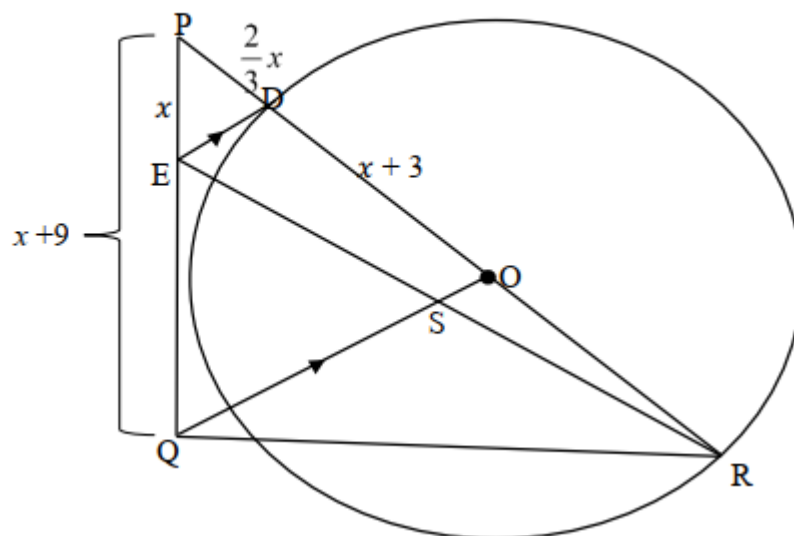
OR / OF	
Let $\hat{F}_1 = \hat{F}_2 = x$	proved / reeds bewys
$\hat{C} = 180^\circ - 2x$	opp $\angle$ s of a cyclic quad / teenoorst $\angle$ e v kvh
$\hat{C}_1 = \hat{C}_2 = 90^\circ - x$	diag rhombus bisect $\angle$ / diag ruit halveer $\angle$
In $\triangle FDC$ or / of $\triangle EFC$	
$\hat{D} = 90^\circ$ or / of $\hat{E} = 90^\circ$	sum of $\angle$ s of $\triangle$ / som van $\angle$ e v $\triangle$
FC is a diameter of circle FDCE.	converse $\angle$ in a semi circle /
FC is 'n middellyn van sirkel FDCE.	omgekeerde $\angle$ in half sirkel

10.2

10.2.1	$\hat{R}_1 = 60^\circ$ $\hat{W}_1 = \hat{P}_1 + \hat{Q}_1$ $= 60^\circ + \hat{Q}_1$ $= \hat{R}_1 + \hat{Q}_1$ $\hat{Q}_1 = \hat{R}_2$ $\therefore \hat{W}_1 = \hat{T}\hat{R}\hat{Q}$	equilateral $\triangle$ / gelyksydige $\triangle$ ext. $\angle$ of a $\triangle$ / buite $\angle$ v $\triangle$ $\angle$ s in the same segment / $\angle$ e in dieselfde segment
10.2.2	In $\triangle TQR$ and / en $\triangle QRV$ 1. $\hat{W}_1 = \hat{T}\hat{R}\hat{Q}$ 2. $\hat{R}_1 = \hat{T}\hat{Q}\hat{R}$ 3. $\hat{Q}_2 = \hat{T}$ $\therefore \triangle WRQ \parallel \triangle RQT$	proved / reeds bewys equilateral $\triangle$ / gelyksydige $\triangle$ sum $\angle$ s of $\triangle$ / som van $\angle$ e v $\triangle$ $\angle \angle \angle$

10.2.3	In $\triangle TPV$ and / en $\triangle WQR$ 1. $\hat{PQR} = \hat{R}_1$ both 60° / albei 60° $\hat{PQR} = \hat{V}_1$ ext. $\angle$ of a cyclicquad. / buite $\angle$ vkvh $\hat{V}_1 = \hat{R}_1$ 2. $\hat{P}_2 = \hat{TRQ}$ ext. $\angle$ of a cyclicquad. / buite $\angle$ vkvh but / maar $\hat{W}_1 = \hat{TRQ}$ proved / reedsbewys $\hat{P}_2 = \hat{W}_1$ 3. $\hat{T} = \hat{Q}_2$ sum of $\angle$s of $\triangle$ / som v $\angle$ ev $\triangle$ $\triangle VPT \parallel \triangle RWQ$ $\angle\angle\angle$ $\frac{VP}{RW} = \frac{PT}{WQ} = \frac{VT}{RQ}$ corresponding sides in prop / ooreenkomstige sye in verhouding $\therefore \frac{PT}{WQ} = \frac{PV}{WR}$
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QUESTION / VRAAG 11



11.1	$\frac{PE}{EQ} = \frac{PD}{DO}$ $\frac{x}{9} = \frac{\frac{2}{3}x}{x+3}$ $x^2 + 3x = 6x$ $x^2 - 3x = 0$ $x(x-3) = 0$ $x = 0 \text{ or/of } x = 3$ N.A / n.v.t $DO = 6$ $DO = OR$ $OR = 6 \text{ units / eenhede}$	line one side $\triangle POQ$ OR prop theorem $ED \parallel OQ$ / lyn een sy $\triangle POQ$ OF eweredigheid stelling $ED \parallel OQ$ radii / radiusse
11.2	S is the midpoint of RE / S is die middelpunt van RE $DE = 2OS$ $DE = 2,8 \text{ units / eenhede}$	midpoint theorem / middelpunt stelling
11.3	$\frac{\text{Area } \triangle PED}{\text{Area } \triangle PER} = \frac{PD}{PR}$ $= \frac{2}{14}$ $= \frac{1}{7}$ $\text{Area } \triangle PER = 7 \times \text{Area } \triangle PED$ $= 18,9 \text{ units}^2 / \text{eenhede}^2$	same height (DE) / dieselfde hoogte (DE)